The Semantic Foundations of Philosophical Analysis

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April Draft

"The business of philosophy, as I conceive it, is essentially that of logical analysis" - Russell

Abstract

I provide an analysis of sentences of the form 'To be F is to be G' in terms of exact truth-maker semantics—an approach which identifies the meanings of sentences with the states of the world directly responsible for their truth-values. Roughly, I argue that these sentences hold just in case that which makes something F is that which makes it G. This approach is hyperintensional, and possesses desirable logical and modal features. In particular, these sentences are reflexive, transitive and symmetric, and, if they are true, then they are necessarily true, and it is necessary that all and only Fs are Gs. I integrate this account with the λ-calculus and argue that analysis is preserved through β-conversion. I close with a brief discussion of how it might be extended to analyses of singular terms.

1   Introduction

The subject of this paper is a targeted reading of sentences of the form 'To be F is to be G,' which philosophers often use to express analyses, and which have occupied a central role in the discipline since its inception. Examples that naturally lend themselves to this reading include:

1. To be morally right is to maximize utility.

2. To be human is to be a rational animal.

3. To be water is to be the chemical compound H_2O.

4. To be even is to be a natural number divisible by two without remainder.

5. To be a béchamel is to be a roux with cream.

As these examples indicate, although philosophers frequently utter these sorts of sentences, they do not fall within the exclusive purview of philosophical inquiry. Mathematicians, chemists, and even chefs use them as well. While some are the sorts of sentences

which, if true, are knowable *a priori*, others are sensitive to empirical investigation—so there is variation within the phenomenon I seek to describe.

Sentences of this form have been employed since antiquity (as witnessed by 2). Throughout the ensuing history, proposed instances have been advanced and rejected for multitudinous reasons. On one understanding, this investigation thus has a long and rich history—perhaps as long and rich as any in philosophy. Nevertheless, explicit discussion of these sentences in their full generality is relatively recent (most notably, see Dorr 2016 and Correia 2017, but also Rayo 2013 and Linnebo 2014). Recent advancements in hyperintensional logic provide the necessary resources to analyze these sentences perspicuously—to provide an analysis of analysis. It is my hope that the significance of this project is apparent; the standards that putative analyses must meet hang in the balance.

A bit loosely, it is my claim that these sentences are true just in case that which makes it the case that something is F also makes it the case that it is G (and vice versa). There is a great deal to say about what I mean by 'makes it the case that.' In some ways, this paper can be read as an explication of that term. For the moment, suffice it to say that rather than understanding it modally (along the lines of 'To be F is to be G' is true just in case the fact that something is F necessitates that it is G and vice versa), I employ truth-maker semantics: an approach which identifies the meanings of sentences with the states of the world exactly responsible for their truth-values.[1] It will take some time before my account can be stated any more precisely; the details of truth-maker semantics must first be appreciated.

Why do I believe that this is true? Two reasons, primarily. The first is that I find it to be extremely intuitive. Although philosophers often trade in intuitions, this virtue remains underappreciated. Many philosophical positions (even many philosophical positions described as 'intuitive') come across as moderately plausible at best, if not clearly incorrect. It is strong a mark in favor of a theory if it strikes those who consider it with intuitive force: as the sort of thing which we should have accepted all along. Strong—but inconclusive. I myself am hit by the intuitive force of naïve set theory, yet I do not believe that it is true. And, all too often, we lack any theory that is so intuitive, and must make do with the best alternative. Nevertheless, an account whose plausibility is manifest, which I take this account to be, garners *prima facie* support.

The second reason I believe that this is true is that it satisfies our theoretical demands with minimal theoretical costs. The applications of the phenomenon I account for are widespread—spanning nearly all branches of philosophy (and beyond). That my account explains its logical and modal features is a mark in its favor. In this vein, I am reminded of the Lewis (1986) doctrine of modal realism: a view he held primarily because of its theoretical power. The ability to account for modality, and those derivative notions that appeal to it, was a weighty payoff. The cost of this power was a rich ontology of possible worlds: a commitment to a robust plethora of entities that exist in precisely the same way

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[1]For the canonical development of truth-maker semantics, see Fine 2013, 2014, 2016, forthcoming *b*.

that the actual world exists. For most contemporary metaphysicians, this cost has proven too heavy to bear. The account I defend has no comparable ontological cost. Of course, this is not to say that it has *no theoretical* costs. But, as we shall see, it is remarkably metaphysically neutral; many of the anti-metaphysical scruples (such as worries about reifying bizarre abstract entities like properties) that philosophers harbor do not apply to this position.

The structure of this paper is as follows. In section 2, I further articulate the targeted reading of 'To be F is to be G' that I address. In section 3, I discuss current developments in truth-maker semantics in considerable detail. I urge the reader to bear with me as I do. The various ways in which the semantics might be constructed bear on philosophical analysis in that the logical features of analysis turn on which alternative we select. For example, the truth-value of sentences of the form 'To be F is to be both F and F' depends on a choice-point in truth-maker semantics—or so I will argue. In section 4, I provide the details of my account and demonstrate that it has the logical and modal features that it ought to. In particular, it is transitive, reflexive and symmetric, and has the resources to distinguish between the meanings of predicates with necessarily identical extensions (sentences of the form 'To be F is to be both F and G or not G' are typically false); further, if a sentence of the form 'To be F is to be G' is true then it is necessarily true, and necessary that all and only Fs are Gs. I integrate this account with the λ-calculus—the predominant method of formalizing logically complex predicates—and argue that analysis is preserved through β-conversion. I provide two methods for expanding this account to address other sorts of analyses, and conclude by defining an irreflexive and asymmetric notion of analysis in terms of the reflexive and symmetric notion.

2   To be F is to be G

'To be F is to be G' may admit of multiple readings. There may well be a reading of 'To be a politician is to be indebted to one's constituents,' 'To be a scientist is to be curious about the natural world,' or 'To be early is to be on time' on which these sentences are true. Perhaps closer to my intended target, there may also be a reading of 'To be a bachelor is to be male' or 'To be crimson is to be red' on which these sentences are true. If there are any such readings, they are not the one that I address. The reading of 'To be F is to be G' that I am concerned with is synonymous with (or, at the very least, close to synonymous with) 'To be F *just* is to be G' or, perhaps, 'being F is *the same as* being G.'

This reading has borne multiple labels in the literature. Some refer to these sentences as 'generalized identities' (e.g., Linnebo (2014), Correia (2017)), others prefer 'identifications' (e.g., Dorr (2016)), while still others refer to them as "just-is'-statements' (e.g., Rayo (2013)). While some labels may be more misleading than others, nothing philosophically significant turns on which label we select—so long as the targeted reading itself is clear.[2]

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[2]For example, it would be understandable if readers erroneously concluded that generalized identities

For the purposes of this paper, I will refer to these sentences as 'generalized identities.' My use of the term 'analysis' differs from 'generalized identity' only in that it is slightly more expansive: it captures both generalized identities and sentences that (may) have different syntactic structures but closely resemble generalized identities.[3]

It is often possible to express analyses using verbs (and verb phrases) in various forms. Examples of these sorts of sentences include:

6. To know that p is to have a justified true belief that p.

7. To die is to cease to live.

8. Shrinking is decreasing in size.

9. To resemble is to be similar to.

It may be that an exhaustive discussion of analysis ought to address these types of sentences as well. However, I presently have very little to say about them, and will largely disregard them here.

However, I maintain that sentences of the form 'For a to be F is for a to be G' belong to the same family as generalized identities—primarily because of their use in expressing analyses of 0-ary predicates. 'To be F is to be G' may, and often does, express analyses of predicates of various adicities. Both 'To be a vixen is to be a female fox' and 'To be adjacent is to be next to' are grammatically correct, although 'vixen' is a unary predicate while 'adjacent' is binary. However, sentences of this form are typically ungrammatical when applied to 0-ary predicates: i.e. predicates that apply to no objects.[4] For example, 'To be John is a bachelor is to be John is an unmarried male' is ungrammatical. However, the sentence, 'For John to be a bachelor is for John to be an unmarried male' is perfectly grammatical, and conveys the information expected of an analysis.

Under the target reading, the 'is' of 'To be F is to be G' shares many logical and modal features with the 'is' of identity. As mentioned above, it is reflexive, symmetric and transitive and, if a sentence of this form is true, then it is necessarily true, and necessary

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are identities—more on this below.

[3]A brief note on my use of the term 'analysis:' Dorr abstains from using this term on the grounds that an account of analysis ought to correspond to an account of analyzing—an epistemic activity that philosophers engage in. I believe that this is mistaken. An analogy may elucidate this mistake. Prior to the Church/Turing developments in computation, computability was taken to be an epistemic property. Computers were people who computed, and a function (or number) was thought to be computable just in case its results could come to be known via a certain method of calculation. We now recognize that computability is not an epistemic property; it is a mathematical property. Of course, it remains possible to generate knowledge by calculation, but computability itself is not defined in epistemic terms. Just so for analysis.

[4]I make the standard assumption that sentences are 0-ary predicates. So, just as 'smaller than' is a binary predicate and 'smaller than Jones' is a unary predicate, 'Smith is smaller than Jones' is a 0-ary predicate.

that all and only Fs are Gs. An adequate account of generalized identities ought, minimally, to explain the presence of these features.

There are some who doubt that 'To be F is to be G' is reflexive and symmetric, maintaining that while 'To be a father is to be a male parent' is true, 'To be a male parent is to be a father' is false, and that 'To be a father is to be a father' is trivially false, rather than trivially true. Such philosophers hold that 'To be F is to be G' forms a strict partial ordering over predicates, perhaps maintaining that if 'To be F is to be G' is true, then G is somehow more basic or fundamental than F is.

I confess that I once had such predilections myself. However, I have come to endorse a reading of 'To be F is to be G' that strongly resembles an identity. I maintain that there is a perfectly intelligible reading of 'To be F is to be F' on which this sentence is manifestly true. After all, what else could F possibly be?[5] Of course, uttering this sort of sentence is typically pragmatically infelicitous. When someone inquires, 'What is it to be a bachelor?' the response 'To be a bachelor is to be a bachelor' is likely to be unhelpful. This can straightforwardly be explained in terms of conversational norms. Because 'To be a bachelor is to be a bachelor' is trivially true, it is not the kind of information such an inquirer is after. Trivial responses to substantive questions flout conversational norms.

Furthermore, interest in one reading need not preclude the import of another. Although I am primarily concerned with the symmetric and reflexive reading of 'To be F is to be G,' it is notable that, on the present approach, it is possible to define an asymmetric and irreflexive notion in terms of the symmetric and reflexive one.[6] So I do not deny that the asymmetric and irreflexive reading exists; instead, I outline an account of such a reading in some concluding remarks, and ask readers primarily interested in this alternate reading to bear with me until then.

It might be tempting to treat 'To be F is to be G' as strictly synonymous with 'To be the property of being F is to be the property of being G', and to treat the 'is' of generalized identities literally as the 'is' of identity. There are several reasons to avoid doing so.[7] These reasons are already quite well rehearsed, but bear revisiting (see Dorr 2016 and Correia 2017).

The first is that it is desirable for an account of 'To be F is to be G' to be agnostic

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[5]Likewise, while there are contexts in which 'To be a father is to be a male parent' seems appropriate, there are also contexts in which 'To be a male parent is to be a father' seems appropriate. See Dorr (2016) for a discussion of this point.

[6]This was largely inspired by Correia (2017), who argues that real definitions are a subset of generalized identities. Roughly, he maintains these are sentences that are true according to the symmetric and reflexive reading, and are such that the term 'F' identifies F-ness *de re* and 'G' limns the metaphysical structure of F. Because the truth of 'To be F is to be G' on the symmetric and reflexive reading is a necessary (but insufficient) condition for the truth of 'To be F is to be G' on Correia's targeted reading, the symmetric and reflexive reading remains significant. Rather than rejecting the reading I am concerned with, those like Correia ought to welcome it, as it plays an integral role in theirs.

[7]However, see Moore (1952) and Carnap (1947) (among others) for discussions of analysis that take the 'is' of analysis to be the 'is' of identity.

with respect to nominalism. Nominalists may grant that a sentence of the form 'To be F is to be G' is true, but would doubtlessly deny that sentences of the form 'The property of being F is the property of being G' are true on the grounds that they deny the existence of properties. This position is, presumably, coherent if nominalism is coherent. While nominalism may (or may not) be tenable, I see no reason to suspect that it is incoherent. Given that nominalism is not guilty of straightforward incoherence, our theory of 'To be F is to be G' ought not entail that nominalism is guilty of straightforward incoherence. In any case, it is important to ascertain how far we can progress in the investigation of analysis without reifying properties. As I hope to demonstrate here, we can progress quite far.[8]

The second reason is that this gloss gives rise to self-referential paradoxes analogous to the Russell paradox. It may be that to be a non-self-instantiator is to not instantiate oneself, but it would be paradoxical to claim that the property of being a non-self-instantiator is the property of not instantiating oneself. If the term 'a non-self-instantiator' (as it figures in 'to be a non-self-instantiator') does not denote, then there is nothing there for the issue of self-application to arise. Paradox can thus be obviated by denying that 'to be a non-self-instantiator' is synonymous with 'to be the property of being a non-self-instantiator' at the outset. To be sure, there are various other ways one could attempt to manage paradoxical self-reference, but it is better to avoid this quagmire in the first place, if at all possible.

It is my hope that the target phenomenon is, by this point, sufficiently clear. I now turn to developments in truth-maker semantics, which I employ in this account.

3   Truth-Maker Semantics

Truth-maker semantics rests upon the conviction that the meanings of sentences directly depend on the aspects of the world that make them either true or false. When stated so generally, this hardly seems controversial.[9] Who would doubt that the meaning of 'grass is green' depends on that which makes it true that grass is green (and that which would make it false that grass is green)? And, to be fair, many accounts can claim conformity to this mantra—at least when it is understood broadly enough. The conviction that sentences' meanings are associated with their truth-conditions has been around at least since Tarski (1933) and arguably since Frege (1892).

What is unique to truth-maker semantics is its commitment to *exact* truth-makers. The aspects of the world responsible for the meaning of a sentence s are those aspects which

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[8]Note that this account is *agnostic*, rather than *committed*, to nominalism. While I harbor nominalist sympathies myself, the account I offer is also compatible with realism. Those who countenance realism may interpret this project as providing identity conditions for propositions, properties and relations.

[9]However, there are alternatives. Davidson (1967) for example, defends a clausal approach such that the meanings of sentences directly depend on their clauses. The immediate truth-makers for the conjunction '$A \wedge B$' are the clauses 'A' and 'B.' They are further parts of language, rather than aspects of the world. I do not have space to do justice to the Davidsonian approach. One reason that I prefer truth-maker semantics is that it provides the resources to prove formal results similar to metatheorems in arithmetic.

exactly make *s* true or *exactly* make *s* false. This distinguishes truth-maker semantics from other approaches—many of which take the meanings of sentences to be determined by objects largely unrelated to their truth-values. The most widespread such account—with advocates ranging from philosophers to linguists to logicians—is Montague semantics (Montague 1970). Montague takes the meaning of a sentence to be a function from possible worlds (and times) to truth-values. So, the meaning of 'snow is white' is a function that takes possible worlds in which snow is white to the True and possible worlds in which snow is not white to the False.

Recourse to possible worlds results in a semantics with both excessive and impoverished amounts of information. On the one hand, the meanings of sentences depend on entire worlds. So the meaning of 'Socrates was Athenian' depends on worlds filled with facts about the height of the Eiffel Tower, the number of stars in the Milky Way Galaxy and the mass of an average pinecone. This is, at best, unintuitive. Why think that the meaning of 'Socrates was Athenian' depends in any way on such heavily saturated objects? On the other hand, and much more worryingly, this approach is incapable of distinguishing between the meanings of necessarily equivalent sentences. If sentence s_1 is necessarily equivalent to sentence s_2, then the sentences are true in precisely the same possible worlds. So, the meaning of '$1 + 1 = 2$' is identical to the meaning of a sentence expressing Goldbach's conjecture (assuming, of course, that Goldbach's conjecture is true). But if the two sentences mean precisely the same thing, how could it be so easy to know that the first is true while it is so difficult to know that the second is?

This problem is especially pressing given our present aim. The target sentences—'To be *F* is to be *G*'—are necessarily true if they are true at all. So, according to Montague semantics, the meaning of any such sentence is a function that takes every possible world to the True. All true generalized identities mean precisely the same thing.

Some attempt to resolve this type of problem metalinguistically (e.g., Stalnaker 1976). In learning that 'To be water is to be the chemical compound H_2O' is true, I thereby learn something about the relationship between the terms 'water' and 'the chemical compound H_2O', and if I learn that 'To be morally right is to maximize utility' I thereby learn something about the relationship between 'morally right' and 'maximize utility.' Because it is a contingent fact that the terms 'water' and 'the chemical compound H_2O' denote the same kind, I thereby learn something substantive. And by learning that 'morally right' denotes the same class of acts as 'utility maximizers,' I learn a contingent fact about the language used to express moral claims.

The main problem with this view is that it is obviously false. Chemists are not engaged in a purely linguistic project when investigating chemical composition. And ever since the fall of ordinary language philosophy (and the wane of the linguistic turn in the analytic tradition more generally), few understand the object of all philosophical analyses to be the meanings of terms. Of course, this is not to say that *no* philosophers concern themselves with what terms mean. There has, for example, been considerable attention devoted to the semantics of the term 'know.' But the subject of many analyses are the objects that

terms denote, rather than the terms themselves. Some philosophers seek an analysis of knowledge, rather than 'knowledge.'[10]

This is not intended to be a knock-down assault on Montague semantics. Rather, I intend to distinguish the present approach from its precursors, and to indicate why it is better suited for the project at hand. By design, truth-maker semantics eliminates aspects of sentences' meanings unrelated to their truth-values, and is capable of attributing distinct meanings to necessarily equivalent sentences. To a very large extent, this paper relies on truth-maker semantics as it was developed by Fine (2013, 2014, 2016, forthcoming *b*).[11] I draw heavily on Fine's contribution. At various points, I introduce slight modifications to best serve the current purpose; I will flag any substantive modifications that I make. Nevertheless, the vast majority of the system is his.

3.1 Pre-Semantic Framework

Integral to the truth-maker approach is the idea of a *state*—a fact-like entity that typically has a more restricted scope than possible worlds have. For example, there is a state of whales being mammals, a state of it raining outside, and a state of 2 being an even number. Additionally, there is a state of there being twelve planets in the solar system, and a state of 2 being an odd number. So, states are not restricted to those that actually obtain, nor even to those that possibly could obtain. Here, my use of 'state' is as metaphysically neutral as possible. I make no assumptions about what ontological kind of thing a state is, or about whether or not states figure in the fundamental building blocks of reality. This level of neutrality allows for diverse applications of the semantics. For example, Fine (forthcoming *a*) outlines a deontic logic in terms of exact truth-makers by treating actions as states. One of the only metaphysical assumptions I make is that states are the sorts of things that are capable of mereological composition. So, the state of a ball being both red and round may be the composite of the state of the ball being red with the state of the ball being round.

This neutrality might seem surprising given our present aim. If I claim, 'To be water is to be the chemical compound H_2O', am I not making a metaphysically substantive assertion? Far from being detrimental, I believe that metaphysical neutrality is beneficial. After all, this does not mean that metaphysics plays no role at the end of the day. Philosophers are free to 'plug in' the states that they deem important, and the present semantics will provide conditions in which a generalized identity is true. Metaphysical neutrality allows for many to appeal to this approach.

Let a *state space* be an ordered pair $<S, \sqsubseteq>$, where S is a set of states and $\sqsubseteq$ is a

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[10]Over the years, there have been numerous attempts to modify Montague semantics in order to avoid this result (e.g., Yablo 2014). These attempts have been only moderately successful. While a few puzzles concerning necessarily equivalences have been resolved, many others remain (see Hawke 2017 for a recent discussion of some of the problems that these revisions face).

[11]For precursors to this semantics, see Van Fraassen (1969) and Hwang and Schubert (1993).

binary relation on S. It is intended for $\sqsubseteq$ to be interpreted as the relation of parthood, such that '$s \sqsubseteq s'$' asserts that state s is a part of state s'. I make the standard assumption that parthood is a partial ordering; i.e. that $\sqsubseteq$ satisfies the following criteria:

    a) Reflexivity: For any state $s \in S$, $s \sqsubseteq s$

    b) Antisymmetry: For any states $s, s' \in S$, if $s \sqsubseteq s'$ and $s' \sqsubseteq s$, then $s = s'$

    c) Transitivity: For any states $s, s', s'' \in S$, if $s \sqsubseteq s'$ and $s' \sqsubseteq s''$, then $s \sqsubseteq s''$

Many state spaces, as defined above, are uninteresting. For example, there are state spaces in which no mereological composition occurs. These are ones in which the extension of $\sqsubseteq$ is restricted to reflexivity; every state is a part of itself, and no state is a part of any other. For the purposes of this paper, let us confine our attention to state spaces that allow for arbitrary fusion. For the most part, this can be achieved by stipulating that any two states within S have a fusion within S. However, this approach fails for infinitely large state-spaces; in these cases, it may be that every two states within S have a fusion within S, but that there are infinitely large collections of states within S that lack a fusion within S. A restriction appropriate for infinitely large state spaces requires a few more definitions.

Let an *upper bound* of $T \subseteq S$ be a state s such that, for all states $t \in T, t \sqsubseteq s$. That is to say, an upper bound of a subset of S is a state which contains—as a part—every state within that subset. Let a *least upper bound* of $T \subseteq S$ be a state s such that s is an upper bound of T and, for all upper bounds s' of T, $s \sqsubseteq s'$. Intuitively, we can think of the least upper bound of T as being the smallest upper bound of T—one which is a part of all upper bounds of T. Provably, if there is a least upper bound of T, then there is a unique least upper bound of T.[12] I denote the least upper bound of T as $\bigsqcup T$.

Let a state space $<S, \sqsubseteq>$ be *complete* just in case every subset $T \subseteq S$ has a least upper bound within S. Complete state spaces are those which are closed under arbitrary fusion. Here, I consider only complete state spaces.

Nothing in the definition of a state space distinguishes those states which are possible from those which are impossible. Given the modal implications of generalized identities, it is valuable to be able to make such a distinction. Let a *modalized state space* be an ordered triple $<S, S^\diamond, \sqsubseteq>$ such that $<S, \sqsubseteq>$ is a complete state space, and $S^\diamond$ is a nonempty subset of S consisting of those states which are possible. I assume that $S^\diamond$ is closed under parthood; every possible state s is such that all of its parts are possible states ($s \in S^\diamond \land s' \sqsubseteq s \to s' \in S^\diamond$).

Modalized state spaces have the resources to define compatibility and incompatibility. Let us say that states s and s' are *compatible* just in case their fusion is a member of $S^\diamond$, and are *incompatible* if it is not.

It is also possible to employ modalized state spaces to define a notion of a possible world.

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[12]Proof: For a state space $< S, \sqsubseteq>$, select an arbitrary $T \subseteq S$. Suppose, for reductio, that T has two least upper bounds—$\bigsqcup T^1$ and $\bigsqcup T^2$. From the definition of 'least upper bound' we have that $\bigsqcup T^1 \sqsubseteq \bigsqcup T^2$ and $\bigsqcup T^2 \sqsubseteq \bigsqcup T^1$. Given antisymmetry, this entails that $\bigsqcup T^1 = \bigsqcup T^2$.

Intuitively, possible worlds are maximally consistent modalized state spaces.[13] For a given modalized state space $<S, S^\diamond, \sqsubseteq>$ let a *subspace* be a modalized state space $<S', S'^\diamond, \sqsubseteq'>$ such that $S' \subset S$, $S^\diamond \subseteq S'^\diamond$, and $\sqsubseteq'$ is a conservative extension of $\sqsubseteq$ defined over S'. A subspace $<S', S'^\diamond, \sqsubseteq'>$ of $<S, S^\diamond, \sqsubseteq>$ is a *possible world* just in case $S^\diamond = S'^\diamond$, $\bigsqcup S' \in S^\diamond$, and every set S'' such that $S' \subset S'' \subseteq S$ is such that $\bigsqcup S'' \notin S^\diamond$.

There are several other requirements that might be imposed on modalized state spaces once states are assigned to sentences. We might, for example, insist that no verifier of a sentence is compatible with a falsifier of that sentence, and that every possible state is compatible with either a verifier or a falsifier of every sentence (which corresponds to *exlusivity* and *exhaustivity* conditions of verification, respectively). However, these assumptions play no role for the current project, so I set them aside.

3.2   First-Order Semantics

For obvious reasons, propositional languages are of limited relevance to claims of the form 'To be F is to be G.' Let us restrict our attention to a language capable of expressing predication.

On some approaches, the end of a semantics is to determine whether or not statements are true or false in a given world. However, on the current approach, the aim is to determine what precisely it is within a world that is responsible for the truth-values of sentences. Thus, verifiers must be relevant to the sentences that they render true, and they must be entirely relevant; they may not contain extraneous information. States are the candidate verifiers and falsifiers. So, the state of the republicans controlling the senate verifies 'The republicans control the senate' and falsifies 'The republicans do not control the senate.'

I do not assume that each sentence has a unique verifier and a unique falsifier. Potentially, sentences could have many. The sentence, 'Either Dante wrote *The Divine Comedy* or Aristotle tutored Alexander the Great' presumably has (at least) two verifiers: the state of Dante having written *The Divine Comedy* and the state of Aristotle having tutored Alexander the Great. But it is important for a sentence's verifiers to guarantee its truth and for its falsifiers to guarantee its falsehood. So the state of roses being red is not a verifier for 'roses are red and violets are blue,' despite its relevance to the truth of the conjunction. Furthermore, and as previously mentioned, verifiers and falsifiers do not contain extraneous information. The state of 2 being both even and prime is not a verifier for 'the number 2 is even,' despite the fact that it guarantees the sentence's truth.[14]

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[13]This is perhaps my largest departure point from Fine, who takes possible worlds to be individual states, rather than state spaces. The current approach is best suited for the aim of demonstrating the modal features of generalized identities. Fine's notion of a possible world is a least upper bound of my notion of a possible world.

[14]Note that this differentiates truth-maker semantics from the approach outlined by Barwise and Perry (1983) and Kratzer (2007) who employ a notion of truth-making such that verifiers need not be wholly relevant to the sentences that they render true.

I assume that there are infinitely many individual objects i_1, i_2, ... Let a language L contain a unique name for each object such that $\underline{i_1}$ denotes i_1, $\underline{i_2}$ denotes i_2, etc.. In addition, let L contain infinitely many predicates $P_1, P_2, ...$, as well as the operators $\neg$, $\wedge$ and $\vee$, defined in the standard way.

Let a model $\boldsymbol{M}$ be an ordered quadruple $<S, \sqsubseteq, I, |\cdot|>$ such that $<S, \sqsubseteq>$ is a complete state space, I is the set of individuals and $|\cdot|$ is a valuation function that takes each n-adic predicate P and each ordered combination of n objects $i_1, i_2, ..., i_n$ to an ordered pair $<T, F>$ where both T and F are subsets of S, with the intended interpretation that T is the set of $P(i_1, i_2, ..., i_n)$'s verifiers, and F is the set of its falsifiers. The semantics is given inductively:

i)$^+$ $\quad s \Vdash P(\underline{i_1}, \underline{i_2}, ..., \underline{i_n})$ iff $s \in |(P(i_1, i_2, ..., i_n))|^T$

i)$^-$ $\quad s \dashv\Vdash P(\underline{i_1}, \underline{i_2}, ..., \underline{i_n})$ iff $s \in |(P(i_1, i_2, ..., i_n))|^F$

ii)$^+$ $\quad s \Vdash \neg A$ iff $s \dashv\Vdash A$

ii)$^-$ $\quad s \dashv\Vdash \neg A$ iff $s \Vdash A$

iii)$^+$ $\quad s \Vdash A \wedge B$ iff for some states t and u, $t \Vdash A$ and $u \Vdash B$ and $s = t \sqcup u$[15]

iii)$^-$ $\quad s \dashv\Vdash A \wedge B$ iff either $s \dashv\Vdash A$ or $s \dashv\Vdash B$

iv)$^+$ $\quad s \Vdash A \vee B$ iff either $s \Vdash A$ or $s \Vdash B$

iv)$^-$ $\quad s \dashv\Vdash A \vee B$ iff for some states t and u, $t \dashv\Vdash A$ and $u \dashv\Vdash B$ and $s = t \sqcup u$

It is my hope that readers find this semantics to be extraordinarily intuitive—perhaps even obviously correct. Nevertheless, it has some surprising results. One of the most unexpected is that a verifier of $A \wedge A$ need not be a verifier of A. Every fusion of two distinct verifiers of A is a verifier of the conjunction $A \wedge A$, but these fusions need not themselves be verifiers of A. I do not find this result particularly problematic. For those who do, the semantics can be readily modified by requiring that verifiers are closed under fusion; the fusion of any two verifiers of A is itself a verifier of A. Clauses iii)$^-$ and iv)$^+$ thus become:

iii)$^{-*}$ $\quad s \dashv\Vdash A \wedge B$ iff $s \dashv\Vdash A$ or $s \dashv\Vdash B$ or $s \dashv\Vdash A \vee B$

iv)$^{+*}$ $\quad s \Vdash A \vee B$ iff $s \Vdash A$ or $s \Vdash B$ or $s \Vdash A \wedge B$

There are at least two ways to extend the semantics to clauses with quantifiers. One utilizes generic objects, such that verifiers of universal statements are generic states (rather than states about particular objects). A verifier for 'All numbers are either even or odd' is the state of a generic number being either even or odd.[16] The semantics I advocate instead is instantial: verifiers of universal and existential statements are states concerning their

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[15]'$\sqcup$' denotes mereological fusion—so '$t \sqcup u$' denotes the fusion of state t with state u.

[16]For a defense of generic objects, see Fine and Tennant 1983.

instances. The introduction of predicates is currently more significant than the introduction of quantifiers is, so I opt for the most easily intelligible approach to quantification. The instantial approach treats the semantics of universal statements like large conjunctions. The meaning of the claim 'Everything is F' is treated similarly to the conjunction of the claims '$F(\underline{i})$', for all $\underline{i}$. More formally, we have:

v)$^+$ $s \Vdash \forall x A(x)$ iff there is a function f from I into S such that $f(i) \Vdash A(\underline{i})$ for each $i \in I$, and $s = \bigsqcup\{f(i) : i \in I\}$

v)$^-$ $s \dashv\!\vert \forall x A(x)$ iff for some $i \in I, s \dashv\!\vert A(\underline{i})$

vi)$^+$ $s \Vdash \exists x A(x)$ iff for some $i \in I, s \Vdash A(\underline{i})$

vi)$^-$ $s \dashv\!\vert \exists x A(x)$ iff there is a function f from I into S such that $f(i) \dashv\!\vert A(\underline{i})$ for each $i \in I$, and $s = \bigsqcup\{f(i) : i \in I\}$

For most purposes, the semantics provided will suffice. However, there are various ways it can (and perhaps ought to) be modified. For a discussion of these potential modifications, see Appendix A.

Exact equivalence can be defined as follows: sentences A and B are *exactly equivalent* just in case their verifiers and falsifiers are identical. Many logically equivalent sentences (from a classical perspective) are not exactly equivalent. For example, $A \wedge \neg A$ need not be exactly equivalent to $B \wedge \neg B$. Verifiers of the first sentence are fusions of a verifier of A with a falsifier of A, while verifiers of the second sentence are fusions of a verifier of B with a falsifier of B. And, as mentioned above, A is not even exactly equivalent to $A \wedge A$ unless appropriate modifications are made. Nevertheless, exact equivalence is distinct from syntactic identity. $A \wedge B$ is exactly equivalent to $B \wedge A$, and A is exactly equivalent to $\neg\neg A$.

Strictly speaking, exact equivalence is defined in relation to a model. Sentences A and B are exactly equivalent relative to model $\boldsymbol{M}$ just in case the verifiers and falsifiers of A and B within $\boldsymbol{M}$ are identical. Depending on the states within $\boldsymbol{M}$, there may be many exactly equivalent sentences relative to $\boldsymbol{M}$ or there may be few. Armed with notion of exact equivalence relativized to a single model, we can also define exact equivalence relative to a class of models—i.e. those that are exactly equivalent in all models that have a certain attribute—or universally—i.e. those which are exactly equivalent in any model.

4   The Semantic Foundations of Analysis

Truth-maker semantics provides the resources to account for 'To be F is to be G' perspicuously: to elucidate what I meant when I claimed that these sentences hold just in case that which makes something F is that which makes something G. A sentence of this form

is true just in case, for any name $\underline{i}$, '$F(\underline{i})$' is exactly equivalent to '$G(\underline{i})$.'[17] 'To be a person is to be bound by the categorical imperative' holds just in case the verifiers and falsifiers of the claim that someone is a person are identical to the verifiers and falsifiers of the claim that she is bound by the categorical imperative, and 'To be morally right is to maximize utility' holds just in case the verifiers and falsifiers of the claim that an act is morally right are identical to the verifiers and falsifiers of the claim that it maximizes utility.

Because exact equivalence is strictly defined in relation to a model, the truth of 'To be F is to be G' is also strictly defined in relation to a model; 'To be F is to be G' is true relative to model $\boldsymbol{M}$ just in case, for any name $\underline{i}$, the verifiers and falsifiers of '$F(\underline{i})$' and '$G(\underline{i})$' within $\boldsymbol{M}$ are identical. I suspect, however, that the universal notion of exact equivalence is best-suited for most philosophical purposes. When a philosopher asserts, 'To be human is to be the rational animal,' the intended scope is, presumably, broad. It would be unsatisfying to discover that which makes Maxwell a human is that which makes Maxwell a rational animal relative to only some models. And so, we may restrict our attention to cases with universal scope. In these cases, 'To be F is to be G' is true if and only if '$F(\underline{i})$' and '$G(\underline{i})$' are exactly equivalent in every model (for any name $\underline{i}$). Unless otherwise specified, I will assume for the remainder of this paper that the scope is universal, and will omit reference to a particular model (although much of what I say applies to relativized cases as well).

I hope that the reader will pardon an autobiographical digression; others may learn from my mistake. I was briefly tempted to believe that sentences of the form 'To be F is to be G' are true just in case '$\forall x F x$' is exactly equivalent to '$\forall x G x$.' This is false. '$\forall x F x$' is exactly equivalent to '$\forall x G x$' just in case the sentences' verifiers and falsifiers are identical. On the current approach, universal statements are treated instantially. That is to say, the claim that everything is F is treated analogously to the conjunction of the claims that each individual is F. Suppose that there were two objects i_1 and i_2 such that a verifier for '$F(i_1)$' was a verifier for '$G(i_2)$,' and a verifier for '$F(i_2)$' was a verifier for '$G(i_1)$.' In this case, the conjunction '$F(i_1) \wedge F(i_2)$' has an identical verifier to '$G(i_1) \wedge G(i_2)$'—the verifier for each is the fusion of two identical states. Nevertheless, it would be absurd to take this to lend support to the truth of 'To be F is to be G.' After all, a state which renders '$F(i_1)$' true is a state which renders '$G(i_2)$,' rather than '$G(i_1)$,' true. It is essential to preserve the verifiers and falsifiers for each instance—and not just for the fusion of their instances.

Some predicates are internally logically complex. For example, 'To be a bachelor is to be an unmarried male' identifies the predicate 'bachelor' with the compound predicate 'unmarried male.' Strictly speaking, this complexity is not captured by the semantics discussed in section 3. The valuation function maps each predicate and ordered sequences of names directly to its sets of verifiers and falsifiers, regardless of the predicate's complexity. There are various ways to formalize logically complex predicates, the most familiar of which

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[17]It is straightforward to generalize this account to apply to predicates of any fixed adicity. For any pair of n-adic predicates F and G, the sentence, 'To be F is to be G' is true just in case, for all collections of n names, $\underline{i_1}$— $\underline{i_n}$, '$F(\underline{i_1}, \underline{i_2}, ..., \underline{i_n})$' is exactly equivalent to '$G(\underline{i_1}, \underline{i_2}, ..., \underline{i_n})$.'

is λ-abstraction.[18] On this method, we bind arbitrarily many free variables within an open sentence in order to form a predicate. The predicate 'bachelor' can thus be represented as '$\lambda x.bachelor(x)$' and the predicate 'larger' can be represented as '$\lambda x, y.larger(x, y)$.'

I maintain that the semantics of internal logical complexity mirrors the semantics of external logical complexity. Indeed, it would border on the bizarre if it did not. Verifiers of the sentence, 'John is an unmarried male' are fusions of a verifier of 'John is unmarried' with a verifier of 'John is male.' And so, on the account that I advance, the truth of, 'To be a bachelor is to be an unmarried male' requires that the verifiers of 'John is a bachelor' be fusions of a verifier of 'John is unmarried' with a verifier of 'John is male.' For those interested in the technical details of the λ-semantics, see Appendix B.

This account accommodates the aforementioned logical features of 'To be F is to be G,' primarily because exact equivalence is an equivalence relation. For any F and any $\underline{i}$, '$F(\underline{i})$' is exactly equivalent to '$F(\underline{i})$,' so sentences of the form, 'To be F is to be F' are universally true. Exact equivalence is symmetric; if '$F(\underline{i})$' is exactly equivalent to '$G(\underline{i})$' then '$G(\underline{i})$' is exactly equivalent to '$F(\underline{i})$.' So, if 'To be F is to be G' is true, then 'To be G is to be F' is true. Furthermore, exact equivalence is transitive. If '$F(\underline{i})$' is exactly equivalent to '$G(\underline{i})$' and '$G(\underline{i})$' is exactly equivalent to '$H(\underline{i})$,' it follows that '$F(\underline{i})$' is exactly equivalent to '$H(\underline{i})$.' So, if 'To be F is to be G' is true and 'To be G is to be H' is true, then 'To be F is to be H' is also true. 'To be F is to be G' is thus reflexive, symmetric and transitive. More specifically, this account entails that sentences of the following forms (which correspond to axioms in a logic of exact equivalence) are universally true:

1. To be $\neg\neg F$ is to be F.

2. To be $F \wedge G$ is to be $G \wedge F$.

3. To be $F \vee G$ is to be $G \vee F$.

4. To be $F \wedge (G \wedge H)$ is to be $(F \wedge G) \wedge H$.

5. To be $F \vee (G \vee H)$ is to be $(F \vee G) \vee H$.

6. To be $\neg(F \wedge G)$ is to be $\neg F \vee \neg G$.

7. To be $\neg(F \vee G)$ is to be $\neg F \wedge \neg G$.

Additionally, inferences of the following forms (which correspond to inferences in a logic of exact equivalence) are universally valid:

8. If to be F is to be G, then to be G is to be F.

9. If to be F is to be G and to be G is to be H, then to be F is to be H.

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[18] λ-abstraction is also employed in some formal theories of properties; the property of *being F* can be identified with the λ-abstract $\lambda x.F(x)$. As before, I take no stand on the existence of properties; my concern is solely the formalization of predicates.

10. If to be F is to be G, then to be $\neg F$ is to be $\neg G$.

11. If to be F is to be G, then to be $F \wedge H$ is to be $G \wedge H$.

12. If to be F is to be G, then to be $F \vee H$ is to be $G \vee H$.

It is worth pausing to consider how plausible these results are. Inference 10, for example, results from adopting a bilateral, rather than a unilateral, notion of exact equivalence (i.e., from insisting that sentences are exactly equivalent just in case they have both identical verifiers and identical falsifiers, rather than merely having identical verifiers).[19] To see why this is so, consider the proposal that a generalized identity is true just in case, for any object i, '$F(\underline{i})$' has identical verifiers to '$G(\underline{i})$' (and omitting the inclusion of falsifiers). Nothing in truth-maker semantics guarantees that sentences with identical verifiers have identical falsifiers, so even if a candidate '$F(\underline{i})$' and '$G(\underline{i})$' have identical verifiers, they may have distinct falsifiers. Negation swaps a sentence's verifiers for its falsifiers, so, in this case, '$\neg F(\underline{i})$' would have distinct verifiers from '$\neg G(\underline{i})$,' and 'To be $\neg F$ is to be $\neg G$' would thereby be false.

An example may highlight how implausible this result is. Suppose that a chemist were to discover that to be water is to be the chemical compound H_2O. It would be extremely bizarre if such a chemist could not infer that to not be water is to not be the chemical compound H_2O (or, on an alternate phrasing, such a chemist could not conclude that to be the absence of water is to be the absence of the chemical compound H_2O). There seems to be no further evidence needed (or even that could be offered) for that conclusion. It one could not infer 'To be $\neg F$ is to be $\neg G$' from 'To be F is to be G', it would indicate that something went seriously wrong with out theorizing of generalized identities.

This account distinguishes between the meanings of logically equivalent predicates, as witnessed by the fact that sentences of the following forms are typically false:

13. To be F is to be $F \wedge (G \vee \neg G)$.

14. To be $F \wedge \neg F$ is to be $G \wedge \neg G$.

Sentences of type 13 are typically false because a verifier of '$F(\underline{i}) \wedge (G(\underline{i}) \vee \neg G(\underline{i}))$' is either the fusion of a verifier of '$F(\underline{i})$' with a verifier of '$G(\underline{i})$', or else the fusion of a verifier of '$F(\underline{i})$' with a verifier of '$\neg G(\underline{i})$.' These fusions need not verify '$F(\underline{i})$.' Similarly, sentences of type 14 are typically false because verifiers of '$F(\underline{i}) \wedge \neg F(\underline{i})$' are fusions of a verifier of '$F(\underline{i})$' with a falsifier of '$F(\underline{i})$', while verifiers of '$G(\underline{i}) \wedge \neg G(\underline{i})$' are fusions of a verifier of '$G(\underline{i})$' with a falsifier of '$G(\underline{i})$.'

Still other logical features depend on controversial ways in which the semantics might be constructed. For example, the truth-value of sentences of the following forms depend on a choice-point in truth-maker semantics:

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[19]This example is not chosen at random. The approach advanced by Correia (2017) does not license inference 10. This, I take it, is a serious cost of his proposal.

15. To be F is to be $F \wedge F$.

16. To be F is to be $F \vee F$.

17. To be $F \wedge (G \vee H)$ is to be $(F \wedge G) \vee (F \wedge H)$.

18. To be $F \vee (G \wedge H)$ is to be $(F \vee G) \wedge (F \vee H)$.

Verifiers of '$F(\underline{i}) \wedge F(\underline{i})$' are fusions of two verifiers of '$F(\underline{i})$.' On the initial construction, this fusion need not itself verify '$F(\underline{i})$', so sentences of the form 'To be F is to be $F \wedge F$' are typically false. Further, a falsifier of '$F(\underline{i}) \vee F(\underline{i})$' is the fusion of two falsifiers of '$F(\underline{i})$.' Because this fusion need not falsify '$F(\underline{i})$,' sentences of the form 'To be F is to be $F \vee F$' are typically false; similar considerations apply to sentences of forms 17 and 18. However, if the modification insuring that verifiers are closed under fusion is introduced, then sentences of forms 15-18 are universally true.

4.1 Haecceitism and Anti-Haecceitism

It might appear that reference to multiple entities is superfluous. If 'Hillary is moving' were exactly equivalent to 'Hillary is at different locations at different times,' it would be odd if 'Amanda is moving' were not exactly equivalent to 'Amanda is at different locations at different times.' After all, the pairs of sentences differ only in the names that they contain. Some might maintain that the exact equivalence of the first pair of sentences ought to guarantee the exact equivalence of the second. If '$F(a)$' is exactly equivalent to '$G(a)$,' one might reasonably think, this occurs due to the relation between the predicates F and G, rather than facts about a. And if the equivalence between 'Hillary is moving' and 'Hillary is at different locations at different times' really does guarantee that, for any name $\underline{i}$, '$\underline{i}$ is moving' is equivalent to '$\underline{i}$ is at different locations at different times', why is this account specified in terms of all names, rather than just one?

Of course, others might disagree. Perhaps differences between names is significant. One way to articulate the point of disagreement is through the following distinction:

a) **Haecceitism:** For some (unary) predicates F and G, there are exist names $\underline{i_n}$ and $\underline{i_m}$ such that '$F(\underline{i_n})$' is exactly equivalent to '$G(\underline{i_n})$', but '$F(\underline{i_m})$' is not exactly equivalent to '$G(\underline{i_m})$.'[20]

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[20]This use of 'haecceitism' differs substantially from what some contemporary philosophers mean by the term. Often, 'haecceitism' is defined as the claim that two possible worlds can differ without differing in their qualitative properties (e.g., Lewis 1986). So, for example, if there is a possible world consisting of two identical spheres, haecceitists contend that there is distinct possible world in which the two spheres switch places. Anti-haecceitists, in contrast, contend that there is no such distinct world. I employ the term not to invoke these sort of modal claims, but to highlight the semantic importance of the 'thisness' of objects according to haecceitism.

b) Anti-Haecceitism: For all (unary) predicates F and G, if there exists a name $\underline{i_n}$ such that '$F(\underline{i_n})$' is exactly equivalent to '$G(\underline{i_n})$,' then, for any name $\underline{i_m}$, '$F(\underline{i_m})$' is exactly equivalent to '$G(\underline{i_m})$.'[21]

I take no stand on whether haecceitism or anti-haecceitism is true. Neither position merits modifying the current account. If the anti-haecceitist is correct, the exact equivalence of 'Napoleon is F' to 'Napoleon is G' entails that 'To be F is to be G' is true. Nevertheless, it would be impermissibly arbitrary to account for 'To be F is to be G' in terms of Napoleon rather than any other individual, and philosophy abhors arbitrary accounts. There is nothing about Napoleon that could, even in principle, justify defining 'To be F is to be G' in terms of him instead of Alexander the Great, Joan of Arc, or anything else (my apologies, Napoleon).[22] Both the haecceitist and anti-haecceitist ought account for these sentences in terms of all objects; the haecceitist because all objects are needed in order to ensure that this account correctly diagnoses the relevant cases, and the anti-haecceitist because arbitrary philosophical decisions are to be avoided. Nevertheless, the anti-haecceitist has an inferential resource that the haecceitist lacks. She may infer that 'To be F is to be G' is true from a single case in which '$F(\underline{i})$' is exactly equivalent to '$G(\underline{i})$.' The haecceitist, in contrast, need first demonstrate the exact equivalence between '$F(\underline{i})$' and '$G(\underline{i})$' for all $\underline{i}$ before concluding that 'To be F is to be G' is true.

4.2 The Modal Implications of Analysis

Although philosophers have largely (and quite rightly) abandoned the idea that analysis can be understood in purely modal terms, it is relatively uncontroversial that analyses have modal implications.[23] In particular, many maintain that if 'To be F is to be G' is

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[21]It is straightforward to articulate the distinction between haecceitism and anti-haecceitism when applied to predicates of any fixed adicity. Haecceitism amounts to the claim that, for some n-adic predicates F and G, there two collections of n names $\underline{i_1}, \underline{i_2}, ..., \underline{i_n}$ and $\underline{i_m}, \underline{i_{m+1}}, ..., \underline{i_{m+n\text{-}1}}$ such that '$F(\underline{i_1}, \underline{i_2}, ..., \underline{i_n})$' is exactly equivalent to '$G(\underline{i_1}, \underline{i_2}, ..., \underline{i_n})$' but '$F(\underline{i_m}, \underline{i_{m+1}}, ..., \underline{i_{m+n\text{-}1}})$' is not exactly equivalent to '$G(\underline{i_m}, \underline{i_{m+1}}, ..., \underline{i_{m+n\text{-}1}})$.' In contrast, anti-haecceitism is the claim that, for any n-adic predicates F and G, if, for any collection of n names $\underline{i_1}, \underline{i_2}, ..., \underline{i_n}$, '$F(\underline{i_1}, \underline{i_2}, ..., \underline{i_n})$' is exactly equivalent to '$G(\underline{i_1}, \underline{i_2}, ..., \underline{i_n})$', then for any other collection of n names $\underline{i_m}, \underline{i_{m+1}}, ..., \underline{i_{m+n\text{-}1}}$ '$F(\underline{i_m}, \underline{i_{m+1}}, ..., \underline{i_{m+n\text{-}1}})$' is exactly equivalent to '$G(\underline{i_m}, \underline{i_{m+1}}, ..., \underline{i_{m+n\text{-}1}})$.'

[22]The anti-haeccieitist might, instead, attempt to account for 'To be F is to be G' existentially; 'To be F is to be G' holds just in case there is some name $\underline{i_n}$ such that '$F(\underline{i_n})$' is exactly equivalent to '$G(\underline{i_n})$.' This avoids accounting for these sorts of sentences in terms of an arbitrary name. And, as far as truth-conditions are concerned, what the anti-haecceitist says is strictly correct. I nevertheless retain my belief that the anti-haecceitist ought to account for 'To be F is to be G' universally. What is of central interest to philosophers are facts that hold of all objects. That the anti-haecceitist can infer universal conclusions from existential premises is a valuable deductive tool, but she remains ultimately concerned with all objects.

[23]It has been suggested that modal accounts of analysis were once "so wide-spread that it would be pointless to provide references" (Correia, 2005, pp. 26). However, see Marcus (1967), Kripke (1980), and Plantinga (1974) for defenses of this claim. This was overturned largely by Fine (1994, 1995a,b), who argued that there are many necessary yet inessential connections between different sorts of things.

true then it is necessarily true, and necessary that all and only Fs are Gs. There are, perhaps, two dominant motivations for accepting these implications. The first concerns the aforementioned analogy between the 'is' of identity and the 'is' of 'To be F is to be G.' Given their strong resemblance, it is reasonable to expect these terms to share not only their logical but also their modal profiles. Philosophers have independently argued that identity holds necessarily (see Kripke 1980). And so, it is plausible that true instances of 'To be F is to be G' hold necessarily as well. The second motivation is that it best reflects philosophical practice. Philosophers often dismiss putative analyses on the basis of possible counterexamples. One need not build an actual paper mâché barn to undermine putative analyses of knowledge that are threatened by barn-facade cases. If 'To be F is to be G' allowed for possible situations in which an F is not a G (or vice versa), this philosophical practice would be unfounded. An account of 'To be F is to be G' which permitted the possibility of Fs that are not Gs (or vice versa) would thus sharply diverge from the way philosophical enquiry is carried out—perhaps so wildly that it would be dubious that any such account captured the notion of analysis we are concerned with.

Recall that a modalized state space is an ordered triple $<S,\ S^{\diamond},\ \sqsubseteq>$ where S is a set of states, $S^{\diamond}$ is a nonempty subset of those states which are possible and $\sqsubseteq$ is a binary relation of parthood on S—and that a possible world is a maximally consistent modalized state space. On this definition, the set of possible worlds is a subset of the set of modalized state spaces. And so, one way to demonstrate that something holds in every possible world is to demonstrate that it holds in all modalized state spaces.[24]

It is straightforward to establish that if 'To be F is to be G' is true then it is necessarily true. Recall that 'To be F is to be G'—as used by philosophers to express analyses—is true just in case it is true relative to all models. If 'To be F is to be G' is true, then, in every model, the states that make something F are those that make it G. If every model is such that the states that make something F are those that make it G, then every modalized state space is such that the states that make something F are those that make it G. After all, the only difference between a modalized state space and a model is that modalized state spaces specify which states are possible (and models to not) and that models specify the set I of individuals and contain a valuation function (which modalized state spaces do not). Because of this, if 'To be F is to be G' is true then it is true relative to all modalized state spaces, and thus to all possible worlds—which form a subset of the modalized state spaces. And so, if 'To be F is to be G' is true, then it is necessarily true.

For similar reasons, if 'To be F is to be G' is true then it is necessary that all and only Fs are Gs. If a sentence of this form is true, then for an arbitrary individual i in an arbitrary possible world W, the states which make i F in W are the same as those which make it G in W. And so, all and only of the Fs in W are Gs. Because the selection of W

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[24]Note, however, that the converse does not hold. Even if 'To be F is to be G' were true relative to all possible worlds, this would not entail that it is true relative to all modalized state spaces. Some modalized state spaces contain impossible states, while no possible worlds contain impossible states. This is plausibly a source of failure of modal accounts of analysis.

was arbitrary, in every possible world all and only Fs are Gs.

Perhaps some suspect that the modal implications of analysis came too easily. I myself am content with easy implications, but it may seem that the definition of possible worlds I employ is illicit. I can already hear an objector exclaiming, 'Play fair! Do not call a thing a 'possible world' unless you demonstrate that it satisfies our pre-theoretic standards for what a possible world is.'

This is a fair and reasonable challenge, but also a somewhat difficult one to discharge at present, in part because there is little consensus on what our pre-theoretic standards for possible worlds are. There are a great many theories of possible worlds. One, which is undoubtedly the most historically significant (if not the most prevalent), is modal realism. Championed by Lewis (1986), modal realism is the view that possible worlds are the same sorts of things as the actual world—physical objects filled with entities distributed throughout space and time. According to another family of views, possible worlds are abstract objects. Perhaps they are maximally consistent sets of propositions (as suggested by Adams (1974)), or else a maximally consistent state of affairs (as suggested by Plantinga (1974, 1976)). Still others adopt something of a middle-ground, maintaining that possible worlds are maximally consistent combinations of metaphysical simples (but diverging from Lewis in maintaining that these simples need not be spatiotemporally located objects). Armstrong (1978, 1986, 1997), for example, builds upon Wittgenstein's contention that the world is the totality of facts (not of things), and argues that a possible world is a maximally consistent fact.

To the extent that I can, I resist the temptation to decide between these competing views. Adopting any one of these positions would make enemies of friends, potentially sacrificing proponents of my account. And this abstention is easy enough. From the outset, I have remained neutral about what kind of thing a state is. Those who disagree about the constituents of possible worlds may each refer to their preferred notion a 'state,' and my account will identify a possible world with a maximally consistent set of states.[25]

It is worth recalling, in this regard, that one of the chief advancements in the formalization of modal logic was the Kripke (1963) realization that possible worlds can be regarded as arbitrary points with accessibility relations defined between them, rather than as particular kinds of entities. Much of modal logic can thus be implemented without taking a stand on what kind of thing a possible world is. My use of 'possible world' is not quite as neutral as Kripke's. I maintain that possible worlds are maximally consistent, and that they are endowed with internal mereological structure. That possible worlds have mereological structure is an assumption I happily take on board, and that they are maximally consistent is a remarkable point of consensus among diverse, competing views. Many notions of possible worlds endorse these assumptions, and any such notion may appeal to my account.

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[25]Or, more precisely, with a modalized state space, the first and second elements of which are maximally consistent sets of states.

4.3 β-Conversion

There is, I believe, a way in which analyses of predicates of varying adicities relate to one another. If, for example, 'To be a sister is to be a female sibling' is true, then 'To be a sister of Jacob is to be a female sibling of Jacob' is also true, and if 'To be white is to have such-and-such a phenomenal character' is true then 'For the Taj Mahal to be white is for the Taj Mahal to have such-and-such a phenomenal character' is also true. In each case, it appears that an analysis of an n-ary predicate guarantees an analysis of an n-1-ary predicate. For the first, a binary analysis was reduced to a unary analysis, and in the second a unary analysis was reduced to a 0-ary analysis.[26]

λ-abstraction provides the resources to reduce the adicity of predicates through a process called β-conversion. If there is a λ-term with n variables (that are bound by the λ-abstract), one of those variables may be replaced by the name of an object. One may, for example, derive the predicate $\lambda x.(x\ is\ married\ to\ Sarah)$ from the predicate $\lambda x, y.(x\ is\ married\ to\ y)$. The relation between the analyses described above can be characterized in the following way: analysis is preserved through β-conversion. If there are two predicates F and G such that 'To be F is to be G' is true, and the predicates F' and G' are the results of performing the same β-conversion (i.e. with the same name on the same variable) on both F and G, then 'To be F' is to be G'' is also true.

Let there be two n-ary predicates F and G, which we identify with the λ-abstracts $\lambda x_1, x_2, ..., x_{\mathrm{n}}.F$ and $\lambda x_1, x_2, ..., x_{\mathrm{n}}.G$ such that 'To be F is to be G' is true. On the account I have advanced, this entails that, for any collection of n names $\underline{i_1}$, $\underline{i_2}$, ... , $\underline{i_n}$, '$F(\underline{i_1}, \underline{i_2}, ..., \underline{i_{\mathrm{n}}})$' is exactly equivalent to '$G(\underline{i_1}, \underline{i_2}, ..., \underline{i_{\mathrm{n}}})$.' Select an arbitrary name $\underline{i_m}$, and let F' and G' be the predicates $\lambda x_1, x_2, ..., x_{\mathrm{v\text{-}1}}, x_{\mathrm{v+1}}, ..., x_{\mathrm{n}}.F[\underline{i_{\mathrm{m}}}/x_{\mathrm{v}}]$ and $\lambda x_1, x_2, ..., x_{\mathrm{v\text{-}1}}, x_{\mathrm{v+1}}, ..., x_{\mathrm{n}}.G[\underline{i_{\mathrm{m}}}/x_{\mathrm{v}}]$—i.e. the results of performing the same β-conversion on both F and G with $\underline{i_m}$ for variable x_v. The above entails that for any collection of $n-1$ names, '$F(\underline{i_1}, \underline{i_2}, ..., \underline{i_{\mathrm{v\text{-}1}}}, \underline{i_{\mathrm{m}}}, \underline{i_{\mathrm{v+1}}}, ..., \underline{i_{\mathrm{n}}})$' is exactly equivalent to '$G(\underline{i_1}, \underline{i_2}, ..., \underline{i_{\mathrm{v\text{-}1}}}, \underline{i_{\mathrm{m}}}, \underline{i_{\mathrm{v+1}}}, ..., \underline{i_{\mathrm{n}}})$' for all collections of $n-1$ names $\underline{i_1}$, $\underline{i_2}$, ..., $\underline{i_{v\text{-}1}}$, $\underline{i_{v+1}}$, ..., $\underline{i_n}$. And so, 'To be F' is to be G'' is true.

Whether the converse holds—i.e. whether or not 'To be F is to be G' follows from 'To be F' is to be G'' where F' and G' are predicates resulting from performing the same β-conversion on F and G—depends on whether haecceitism or anti-haecceitism is correct. If the haecceitist is correct, it may be that 'for Joan to be a vixen is for Joan to be a female fox' is true, while 'for Sarah to be a vixen is for Sarah to be a female fox' is false. If this were so, 'To be a vixen is to be a female fox' would be false (due to the difference in verifiers and falsifiers involving Sarah) despite the fact that 'for Joan to be a vixen is for Joan to be a female fox' is true, and is the result of β-converting both the predicates 'vixen' and 'female fox' with the name 'Joan.' If, instead, the anti-haecceitist is correct, the exact equivalence between 'Joan is a vixen' and 'Joan is a female fox' guarantees that, for any name $\underline{i}$, '$\underline{i}$ is a

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[26] Although this inference is extremely natural in most cases, it arguably fails for languages that contain opaque terms like 'believes.' For the purposes of this paper, I restrict my attention to languages that lack these sorts of terms.

vixen' is exactly equivalent to 'i' is a female fox—so it follows from 'for Joan to be a vixen is for Joan to be a female fox' that 'To be a vixen is to be a female fox.' Indeed, one way to characterize the difference between the haecceitist and the anti-haecceitist is with respect to whether or not analysis is preserved through inverse β-conversion.

4.4 Analysis and Singular Terms

Over the years, philosophers have provided analyses that this account, arguably, does not address. Perhaps some would claim '{Socrates} is the set containing only Socrates' or 'The universe is the mereological composite of all objects' express analyses. '{Socrates}' and 'The universe' are names, and so this account—which is directly addresses only predicates—may seem inapplicable to these cases.

There are two methods of expanding the present account. On one approach, we might consider an analysis concerning '{Socrates}' as an analysis concerning '={Socrates}.[27] The sentence 'To be {Socrates} is to be the set containing only Socrates' may be translated to 'To be identical to {Socrates} is to be identical to the set containing only Socrates.' On the present approach, this sentence is true just in case the states of the world which make it the case that something is identical to {Socrates} are those which make it the case that it is identical to the set containing only Socrates. In general, an analysis concerning a singular term 'a' is translated into an analysis concerning the predicate '$= a$', before the previous account of analyses of predicates is applied. This approach takes seriously the claim that an analysis of a thing is the identity of that thing; an analysis concerning singular terms specifies what it takes to be identical to the objects denoted by those terms.

But perhaps some take this step to be misguided, preferring an analysis of {Socrates} rather than an analysis of what it takes to be identical to {Socrates}. Such philosophers would resist appealing to this method. A second method appeals to the view that proper names are predicates. To be clear, I am not myself committed to the claim that proper names are predicates. But if they are (and the position that they are has grown increasingly influential), the present account applies to further cases.

The connection between names and predicates can be traced to the Frege/Russell discussion of proper names and propositional attitudes. Many attribute to the pair the view that proper names are disguised definite descriptions (Russell 1905). The name 'Aristotle', for example, may mean 'the pupil of Plato and teacher of Alexander the Great.' This view has largely fallen out of favor, but more recent (and sophisticated) positions that treat names as predicates have arisen in its place. Quine (1948), for example, treats the meaning of 'Pegasus' as 'that which pegasizes.' Quine's motivations for converting names into predicates are primarily philosophical. He maintains that ontological issues—such as whether or not Pegasus exists—ought not be settled by semantic considerations. Those who deny that Pegasus exists can do so without apparent contradiction if the sentence 'Pegasus does not exist' means 'There does not exist an object that pegasizes.'

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[27]This is extremely similar (although more metaphysically neutral) the approach outlined by Fine (2015).

More recently, philosophers have argued that proper names are predicates on linguistic grounds. Burge (1973) argues that proper names are predicates preceded by an omitted demonstrative (so the sentence 'Joan is tall' means 'That Joan is tall' in which the 'that' has been omitted), and Fara (2015) argues that proper names are predicates preceded by an omitted 'the' (so the sentence 'Joan is tall' means 'The Joan is tall' with an omitted 'the'). There are several motivations for these positions. One concerns sentences in which proper names clearly function predicatively. If, for example, I were to meet several people named Sarah at a restaurant, I might say, 'The Sarah's from the conference are running late' (perhaps implying that at least one person named Sarah is not from the conference and is not running late). Given that there are some cases in which proper names are predicates, a unified account treats all proper names as predicates.

If names are predicates, then analyses of names are also analyses of predicates. On the present account, the sentence 'To be {Socrates} is to be the set containing only Socrates' expresses the claim that, for any object o, '{Socrates}(o)' is exactly equivalent to 'the set containing only Socrates(o).' As I have previously said, I do not commit myself to this view of the meanings of names. Nevertheless, those who do have the resources to apply the present framework to a wide array of additional analyses.

5 Conclusion

The predominant focus of this paper has been a reflexive and symmetric notion of analysis: one that closely resembles an identity. However, I have briefly mentioned (and fully endorse) a reading of 'To be F is to be G' that is irreflexive and asymmetric. On this alternate reading, the 'is' of 'To be F is to be G' does not resemble the 'is' of identity; sentences of the form 'To be F is to be F' are universally false, and if 'To be F is to be G' is true, then to be 'G is to be F' is false. Any adequate account of such a reading warrants a paper of its own. However, I take it to be a virtue of this account that it is possible to define the irreflexive and asymmetric notion of analysis (which, instead of a 'generalized identity,' we might call a 'definition') in terms of the reflexive and symmetric notion. Before closing, allow me to briefly sketch this reading here.

The underlying motivation for this approach comes from the idea that a definition of a phenomenon breaks it down into its constituent parts. While some occasionally may say this somewhat metaphorically, I propose an interpretation that is almost literal. Recall that states that verify and falsify sentences are endowed with mereological structure: some of these states have proper parts. The state of John being a bachelor, for example, may be the composite of the state of John being male with the state of John being unmarried. And so, one can learn about the mereological structure of the state of John being a bachelor by understanding what states are parts of it.

A natural thought is that a definition is a sentence of the form 'To be F is to be G' which satisfies three conditions:

1. 'To be F is to be G' is true under the generalized identity reading.

2. 'G' limns the mereological structure of the states that make something F.

3. 'F' identities the states that make something F *de re* (or, if you prefer, 'F' does not limn the mereological structure of the states that make something F).[28]

'To be a mother is to be a female parent' is true, on this reading, just in case the states of the world that make someone a mother are identical to those that make her a female parent, 'female parent' limns the mereological structure of the states that make someone a mother (presumably, by demonstrating that they are fusions of the state of being female with the state of being a parent), and 'mother' does not limn the mereological structure of such states.

It should be clear that account is both irreflexive and asymmetric. Any candidate 'F' and 'G' that satisfy conditions 1-3 are such that 'To be F is to be F' is false (because 'F' fails to limn the mereological structure of the states that make something F—as specified by condition 2), and 'To be G is to be F' is false (both because 'F' fails to limn the mereological structure of the states that make something G—as specified by condition 2 and because 'G' limns the mereological structure of the states that make something G—as specified by condition 3).

It should also be clear that this approach does not restrict the cognitive import of definitions to facts about language. When one learns that 'To be F is to be G' is true, one learns more than facts about the predicates 'F' and 'G'; in particular, one learns about the mereological structures of the states themselves. And so, definitions can convey information about the world.

I believe that this account of definition is worthy of serious consideration, but will say no more about it here. Furthermore, there are advantages to my account of generalized identities that I have failed to address. In particular, it offers insight into the new riddle of induction, the paradox of analysis, and semantic puzzles such as Frege and Mates' puzzles. However, I do not rely on arguments that I lack the space to provide, but hope that the discussion I have offered motivates a conception of analysis in terms of truth-maker semantics, and highlights some of the immediate logical and modal implications of that position.

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[28] As previously mentioned, I am indebted to the discussion of definition in Correia (2017) for this point. The main difference between our proposals is that Correia cashes out conditions 2 and 3 in terms of grounding (or, alternatively, in terms of relative naturalness). I take this approach to be preferable to Correia's in that it provides a more unified account. Why would one condition of analysis concern the mereology of truth-making, while the other conditions employ entirely different notions? On my proposal, the mereology of truth-making underlies every condition of a definition.

Acknowledgements

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Appendix A

Some may find v)⁻ and vi)⁺ inordinately restrictive. The only verifiers of the claim that an F exists are verifiers of the claim that an individual is F. Consider the state of two individuals being F (i.e. the state which is a fusion of two states, each of which is a verifier of the claim that an individual is F). A quite natural English translation of '$\exists x F(x)$' is 'there is at least one F.' So, some may wonder, why would the state of two (or more) individuals being F not be the sort of thing which makes the existential statement true? For those who think along these lines, the semantics given above can be modified in the following way:

v)⁻* $s \dashv\Vdash \forall x A(x)$ iff for some nonempty $J \subseteq I$, there is a function f from J into S
        such that $f(i) \dashv\Vdash A(\underline{i})$ and $s = \bigsqcup\{f(i) : i \in J\}$

vi)⁺* $s \Vdash \exists x A(x)$ iff for some nonempty $J \subseteq I$, there is a function f from J into S
        such that $f(i) \Vdash A(\underline{i})$ and $s = \bigsqcup\{f(i) : i \in J\}$

Some might object to this modification on the grounds that these states overdetermine the truth of existential statements. That is to say, the states contain, as proper parts, states which themselves are verifiers of the existential claims. This objection is misguided; we already allow verifiers to have proper parts that are themselves verifiers. Some verifiers of the conjunction '$A \wedge A$' are the fusions of two distinct verifiers of A. Each of these is itself a verifier of '$A \wedge A$' (because the fusion of a verifier s of A with itself is simply s), so some verifiers of the conjunction overdetermine its truth. All that we require of verifiers is that they entail the truth of the sentences that they make true, and that they lack irrelevant information. The states under consideration satisfy both criteria. That two individuals are F guarantees that '$\exists x F(x)$' is true, and no part of that state is irrelevant to the existential claim.

There is a well-known problem with treating universal statements like large conjunctions, namely that the truth of the conjunction does not necessitate the truth of the universal statement. After all, there are possible situations in which the conjunction is true and the universal statement is not. Suppose that there are three cats in the world and that all three have fur. There is a possible situation in which those three cats have fur and yet 'all cats have fur' is false because, in that situation, there exists a fourth cat who does not have fur.

There are various possible responses to this problem. The first, which does not require modifying the current semantics in any way, is to insist that everything exists necessarily (a la Williamson 2013). If every possible object is an actual object, we need not consider situations in which merely possible objects exist. Another possibility is to interpret the actualist quantifier in terms of the possibilist quantifier $\prod$. If we allow the set I to include not only those entities that actually exist but also those which merely possibly exist, we may interpret the sentence '$\forall x F(x)$' to mean '$\prod x(\neg \exists x \vee F(x))$.' This solution is disingenuous on the truth-maker approach. The fact that a merely possible object does not exist would be part of a verifier of the claim that everything is F. A third approach, and the one I

adopt here, is to include a totality condition. Part of what makes it the case that '$\forall x F(x)$' is true is the fact that the actual objects are precisely the objects that exist.

Let us revise our definition of a model such that a model $\boldsymbol{M}$ is an ordered quintuple $<S, \sqsubseteq, I, \tau, |\cdot|>$. As before, $<S, \sqsubseteq>$ is a complete state space and I is the set of individuals (allowing for I to contain all possible—not merely all actual—individuals).[29] τ is a function taking each subset of I into a state $s \in S$, intuitively the state in which precisely those individuals exist. Clauses v) $^+$—vi)$^-$ can be modified to the following:

v)$^{+**}$ $s \Vdash \forall x A(x)$ iff for some $J \subseteq I$, there is a function f from J into S such that $f(i) \Vdash A(\underline{i})$ for each $i \in J$, and $s = \tau_J \sqcup \bigsqcup\{f(i) : i \in J\}$

v)$^{-**}$ $s \dashv\Vdash \forall x A(x)$ iff for some $J \subseteq I$, $j \in J$, $t \in S$, $t \dashv\Vdash A(\underline{j})$ and $s = t \sqcup \tau_J$

vi)$^{+**}$ $s \Vdash \exists x A(x)$ iff for some $J \subseteq I$, $j \in J$, $t \in S$, $t \Vdash A(\underline{j})$ and $s = t \sqcup \tau_J$

vi)$^{-**}$ $s \dashv\Vdash \exists x A(x)$ iff for some $J \subseteq I$, there is a function f from J into S such that $f(i) \dashv\Vdash A(\underline{i})$ for each $i \in J$, and $s = \tau_J \sqcup \bigsqcup\{f(i) : i \in J\}$

Quine (1954) noted that classical quantifiers have existential import—first order logic holds only if at least one object exists. Arguing that logic ought to be ontologically neutral, he defends the use of inclusive quantifiers; ones that lack existential import. If, given the semantics above, we allow J to be empty, the resulting quantifiers are inclusive. If, instead, we restrict J to nonempty subsets of I, the resulting quantifiers are noninclusive.[30]

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[29]Note that this modification somewhat mollifies the assumption that there are infinitely many entities within I. This assumption now only commits us to the claim that there are infinitely many entities which could possibly exist—not that there are infinitely many that actually exist.

[30]Fine defines inclusive quantification differently.

Appendix B

The following is an account of atomic sentences with unary predicates, where P is an atomic formula (i.e. one without internal logical structure) with one free variable v, and ϕ and ψ are both well-formed formulae each containing one free variable v_1 and v_2 respectively:

i)$^{o+}$ $s \Vdash \lambda x.[P(x/v)](\underline{i_n})$ iff $s \in |(P(i_n))|^T$

i)$^{o\text{-}}$ $s \dashv\!\vert \lambda x.[P(x/v)](\underline{i_n})$ iff $s \in |(P(i_n))|^F$

i)$^{\neg+}$ $s \Vdash \lambda x.[\neg\phi(x/v_1)](\underline{i_n})$ iff $s \dashv\!\vert \lambda x.[\phi(x/v_1)](\underline{i_n})$

i)$^{\neg\text{-}}$ $s \dashv\!\vert \lambda x.[\neg\phi(x/v_1)](\underline{i_n})$ iff $s \Vdash \lambda x.[\phi(x/v_1)](\underline{i_n})$

i)$^{\wedge+}$ $s \Vdash \lambda x.[\phi(x/v_1) \wedge \psi(x/v_2)](\underline{i_n})$ iff there exist states t and u such that
$t \Vdash \lambda x.[\phi(x/v_1)](\underline{i_n})$ and $u \Vdash \lambda x.[\psi(x/v_2)](\underline{i_n})$ and $s = t \sqcup u$.

i)$^{\wedge\text{-}}$ $s \dashv\!\vert \lambda x.[\phi(x/v_1) \wedge \psi(x/v_2)](\underline{i_n})$ iff either $s \dashv\!\vert \lambda x.[\phi(x/v_1)](\underline{i_n})$ or
$s \dashv\!\vert \lambda x.[\psi(x/v_2)](\underline{i_n})$

i)$^{\vee+}$ $s \Vdash \lambda x.[\phi(x/v_1) \vee \psi(x/v_2)](\underline{i_n})$ iff either $s \Vdash \lambda x.[\phi(x/v_1)](\underline{i_n})$ or
$s \Vdash \lambda x.[\psi(x/v_2)](\underline{i_n})$

i)$^{\vee\text{-}}$ $s \dashv\!\vert \lambda x.[\phi(x/v_1) \vee \psi(x/v_2)](\underline{i_n})$ iff there exist states t and u such that
$t \dashv\!\vert \lambda x.[\phi(x/v_1)](\underline{i_n})$ and $u \dashv\!\vert \lambda x.[\psi(x/v_2)](\underline{i_n})$ and $s = t \sqcup u$.

There remains a use for predicates with internal quantification. The predicate 'married', as it occurs in English, functions either as a unary or a binary predicate. We may say 'Alex and Sarah are married' in order to describe a relationship between Alex and Sarah, or, alternatively, may say 'Alex is married.' Plausibly, the unary predicate is analyzed in terms of the binary one. To be married (in the unary sense) is for there to exist a person that one is married to. If this is correct, then the internal structure of the unary predicate 'married' involves existential quantification. Similarly, relationalists about mass contend that to have mass is to stand in a mass relation to another object. That is, they identify the predicate 'has mass' with a predicate (roughly) 'being such that there exists another object that it is either as massive, more massive or less massive than.' So the relationalist requires a predicate with internal existential quantification. We have a use, then, for predicates with internal quantifiers.

Let ϕ be a well-formed formula that contains two free variables—v_1 and v_2. The semantics of atomic sentences containing unary predicates with internal quantification is:

i)$^{\forall+}$ $s \Vdash \lambda x.[\forall y \phi(x/v_1)(y/v_2)](\underline{i_n})$ iff there is a function f from I into S such that
$f(i_m) \Vdash \lambda x.[\phi(x/v_1)(\underline{i_m}/v_2)](\underline{i_n})$ for each $i_m \in I$, and $s = \bigsqcup \{f(i) : i \in I\}$

i)$^{\forall\text{-}}$ $s \dashv\!\vert \lambda x.[\forall y \phi(x/v_1)(y/v_2)](\underline{i_n})$ iff for some $i_m \in I$, $s \dashv\!\vert \lambda x.[\phi(x/v_1)(\underline{i_m}/v_2)](\underline{i_n})$

i)$^{\exists+}$ $s \Vdash \lambda x.[\exists y \phi(x/v_1)(y/v_2)](\underline{i_n})$ iff for some $i_m \in I$, $s \Vdash \lambda x.[\phi(x/v_1)(\underline{i_m}/v_2)](\underline{i_n})$

i)$^{\exists\text{-}}$ $s \dashv\vdash \lambda x.[\exists y \phi(x/v_1)(y/v_2)](\underline{i_\text{n}})$ iff there is a function f from I into S such that $f(i_\text{m}) \dashv\vdash \lambda x.[\phi(x/v_1)(\underline{i_\text{m}}/v_2)](\underline{i_\text{n}})$ for each $i_m \in I$, and $s = \bigsqcup\{f(i) : i \in I\}$

There are various ways in which this semantics could be modified that are analogous to the modifications discussed in Appendix A. And it is conceptually straightforward (although technically cumbersome) to extend the semantics to describe predicates of any fixed adicity. One complication which arises concerns the fact that the formulas ϕ and ψ that are subject to combination within a predicate may not have the same number of free variables. However, I will spare the reader (any more) excessive formalisms, and leave that extension as an exercise.