Does Definition Admit of Substitution?

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Abstract

I argue that definitional substitution engenders reflexive definitions: cases in which an entity is defined directly and exclusively in terms of itself. I consider two prominent accounts of definition—one according to which definition is a relation between properties and structured complexes, and the other according to which definition is a species of identity—and demonstrate that substitution principles engender reflexive definitions for each. Along the way, I demonstrate that the claims in 'Real Definition' (Rosen (2015)) are logically inconsistent. I close with a brief discussions of the implications this has for the opacity of definition and for philosophical methodology more generally.

1 Introduction

Real definition plays an integral role not only in metaphysics, but in nearly all branches of philosophy.[1] Some moral disputes can be understood as a disagreement over the definition of the good; attempted amendments to a justified-true-belief analysis of knowledge as a search for the definition of knowledge; and one type of physicalism as the claim that everything is defined in purely physical terms. Many philosophers are concerned with the identity of things—with what it is that makes them the entities that they are. The nature of definition is important, at least in part, because it directly impacts much of philosophical inquiry.

This paper concerns the logical features of definition—with what follows from definitional claims. A bit loosely, I am concerned with whether definitions can be *substituted* within one another. Some may suspect that definition admits of substitution. If water is defined in terms of its atomic constituents, and its atomic constituents are defined in terms of their subatomic parts, it seems reasonable to maintain that water is ultimately defined in terms of its subatomic parts. Within the definition of water, the thought goes, water's

[1]By 'real definition' I mean the metaphysical analogue of nominal definition. While the nominal definition of 'water' is a definition of the term, the real definition of water is a definition of the compound itself. While some dispute the distinction between real and nominal definition, I assume that it is coherent for the purposes of this paper. Here, I use 'definition' to refer to real definition unless otherwise specified.

atomic constituents can be replaced by their own definition. This has nothing to do with water in particular. Rather, it is a logical feature of definition; any time one entity is defined in terms of another, the latter can be replaced with its own definition within the definition of the former. So, if the property of *being a bachelor* is—by definition—the property of *being an unmarried male* and the property of *being unmarried* is—by definition—the property of *lacking a marriage*, then the property of *being a bachelor* is—by definition—the property of *being a male who lacks a marriage*. And if {2} is— by definition—-the set containing only the number 2 and the number 2 is—by definition—the successor to the number 1, then {2} is—by definition—the set containing only the successor to the number 1.

I maintain that this is false; real definition does not admit of substitution. I do not claim that there are no cases in which substitution succeeds. Rather, I claim that even modest substitution principles entail that there are reflexive definitions—cases in which an entity is defined directly and exclusively in terms of itself. I reject reflexive definitions, and suspect that I am in good company in doing so. Strange as the literature on personal identity may be, it has never been suggested that Socrates' definition is Socrates himself. And while some argue that knowledge is primitive, no one has suggested that knowledge is—by definition—knowledge. However, I have no new argument against reflexive definitions at present, so all that I take to demonstrate is a conditional: *if* definition admits of substitution, *then* there are reflexive definitions (or, alternatively, that *either* definition does not admit of substitution, *or* there are reflexive definitions). I am personally more averse to reflexive definitions than wed to substitution principles, so I take this to constitute a reason to abandon substitution principles. Those who do not mind reflexive definitions, however, may retain substitution principles.

Of course, on one interpretation of 'substitution,' this may be fairly unsurprising. Suppose that we license the following substitution principle (without yet taking a firm stand on what definition itself consists of):

1. A is, by definition, B
2. C is, by definition, D

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3. A is, by definition, $B^{C/D}$

If A is, by definition B, and C is, by definition D, then any replacement of C with D within 'A is, by definition, B' is valid. It is easy to see that this accommodates the examples mentioned above. For instance, if we allow sentence 1 to be '{2} is, by definition, the set containing only the number 2' and allow sentence 2 to be 'the number 2 is, by definition, the successor to the number 1,' this principle licenses '{2} is, by definition, the set containing only the successor to the number 1.'

However, this principle quickly generates reflexive definitions. In fact, it generates reflexive definitions so quickly that I have never seen anyone actually endorse it. Suppose, for example, that to be a vixen is, by definition, to be a female fox. If we allow 'To be a vixen is, by definition, to be a female fox' to be both sentences 1 and 2, then the substitution principle licenses 'To be a female fox is, by definition, to be a female fox.' Of course, this is not peculiar to female foxes. Quite generally, by allowing sentences 1 and 2 to be the same, it is possible to generate a reflexive definition.

Presumably for this reason, philosophers have preferred more restricted substitution principles.[2] Reflexivity arose, some might reasonably suspect, because the principle licensed substitution within the definiendum—or the object of analysis. If substitution principles were restricted to the definiens—or the content of analysis, then reflexivity could be avoided.

What I take to be surprising is that these more restricted substitution principles *also* yield reflexive definitions. Here, I explore the two conceptions of definition that I take to be dominant. The first, exemplified by Rosen (2015), is that real definition is a relation between a property and a structured complex. For example, it may be a relation that holds between the property *being a father* and the complex of being a male parent. The second, exemplified by Correia (2017), is that real definitions are a kind of identity.[3] For example, it may be that 'To be even is to be a natural number divisible by 2 without remainder' expresses a definition. On each conception, restricted substitution principles give rise to reflexive definitions, so the link between substitution and reflexivity does not turn on which account of definition we adopt.

I do not canvass everything that might reasonably count as a substitution principle. Perhaps some do not engender reflexive definitions. Nevertheless, I take it that the principles I address are reasonably called 'substitution principles,' and, I suspect, are what many philosophers have in mind when discussing definitional substitution.

I precede as follows. In section 2, I present the view that definition is a relation between properties and structured complexes. In section 3, I demonstrate that definitional substitution gives rise to reflexive definitions on this view. As a corollary, I demonstrate that the claims in Rosen (2015) are logically inconsistent. In section 4, I present the view that definition is a species of identity, and show how substitution principles give rise to reflexive definitions on this view in section 5. I conclude in section 6 by briefly discussing the opacity of definition and the implications this has for philosophical methodology.

2 Definition as a Relation

On one conception, definition is a relation. In particular, Rosen maintains that definition is a relation between properties (and relations) and structured complexes. Rosen provides

[2]For those who endorse substitution in this area, see, e.g., Rosen (2015); Dorr (2016); Correia and Skiles (Forthcoming).

[3]More precisely, definitions are a species of sentences that closely resemble identities without the ontological commitment that the terms flanking the identity sign denote.

no precise account of what a structured complex consists of, saying that it is "built from worldly items in roughly the sense in which a sentence is built from words" (Rosen, 2015, pg. 190). Structured complexes, on this view, resemble Russellian propositions except for the presence of free variables that correspond to the adicity of the property or relation being defined. For example, if being a person is, by definition, being a rational animal, then the definition relation obtains between the property *being a person* and the complex *being a rational animal*.

Problems may already arise. If structured complexes so closely resemble structured propositions, they might inherit obstacles that structured propositions face. For example, Goodman (2017) recently argued that Russell's paradox threatens many conceptions of structured propositions. It may be difficult to characterize the notion of a structured complex in a consistent way. For the moment, let us set this type of worry aside.

Notably, Rosen's account does not require that definitions are identities. Even if *being a mother* is defined in terms of the complex of being a female parent, the two may be distinct from one another. If the relevant structured complexes are not themselves properties or relations, an application of Leibniz's law ensures that a definiendum is distinct from its definiens; after all, the defindum bears the property *being a property or relation*, while the definiens does not.

Rosen's account employs two notational elements that surpass traditional second-order logic. The first is the indexed modal operator $\Box_F$, which appeared first in Fine (1995). Roughly, this can be translated to 'it is necessary in virtue of the identity of F,' so '$\Box_{Knowledge}$(Knowledge is a mental state)' means that it is necessary in virtue of the identity of knowledge that knowledge is a mental state.

The second is a symbol for *grounding*—a relation of metaphysical dependence. Rosen takes grounding to be a relation obtaining between facts, and symbolizes it with the arrow '$\leftarrow$,' so '$A \vee B \leftarrow A$' means that the fact that A or B is grounded in (or obtains in virtue of) the fact that A. Additionally, Rosen makes the standard (but not uncontroversial) assumption that ground is a strict partial ordering, i.e., that the following conditions on grounding obtain:

> 1. Irreflexivity: $\neg(A \leftarrow A)$ (no fact grounds itself).
> 2. Asymmetry: $(A \leftarrow B) \rightarrow \neg(B \leftarrow A)$ (if one fact grounds a second, then the second does not ground the first).
> 3. Transivity: $(A \leftarrow B) \wedge (B \leftarrow C) \rightarrow (A \leftarrow C)$ (if one fact grounds a second, and the second grounds a third, then the first grounds the third. (pg. 201)

Armed with these notions, Rosen provides the following account of definition:

$$Def(F, \phi) \text{ iff } \Box_{\mathrm{F}} \forall x((Fx \vee \phi x) \rightarrow (Fx \leftarrow \phi x))$$

This states that a property F is, by definition, the structured complex ϕ just in case it is necessary in virtue of the identity of F that, for any object, if that object is either F or

ϕ, then it is F in virtue of being ϕ (or, rather, the fact that the object is F is grounded in the fact that it is ϕ). For example, the property of *being morally right* is defined in terms of maximizing utility just in case it is necessary in virtue of the identity of *being morally right* that if an act is either morally right or maximizes utility, then the fact that it maximizes utility grounds the fact that it is morally right.

Although Rosen states his account biconditionally, he intends something stronger. The definition relation itself is a candidate relation subject to definition. Rather than merely providing necessary and sufficient conditions for real definition, Rosen takes himself to have provided a *definition* of definition.

3 From Substitution to Reflexivity (Part 1)

Rosen licenses definitional substitution remarkably explicitly, asserting:

> It should be possible to prove a principle that licenses arbitrary definitional expansion: Def(F, ϕ) and Def(G, ψ) then Def(F, $\phi^{\psi/G}$) where $\phi^{\psi/G}$ is the result of substituting ψ for G in ϕ...Any account of real definition should license the substitutions of definiens for definiendum in a ground to yield a further ground. (Rosen, 2015, pg. 201)

Rosen restricts definitional substitution to the definiens; it is admissible only the content of analysis, not the object of analysis. In particular, he licenses the following substitution principle:

4. Def(F, ϕ)
5. Def(G, ψ)

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6. Def(F, $\phi^{\psi/G}$)

I maintain that this principle is invalid—it sometimes yields false conclusions. To wit, this principle contradicts other claims Rosen makes. In particular, the following claims Rosen makes are logically inconsistent:

a) Grounding is irreflexive
b) The substitution principle
c) Rosen's definition of definition

The inconsistency arises when Rosen's account is applied to itself: when the definition relation is itself subject to definition. Demonstrating this quickly becomes technically cumbersome, so I help myself to the following abbreviation:

$$\mu =_{def} \Box_{\text{F}}\ \forall x((Fx \vee \phi x) \rightarrow (Fx \leftarrow \phi x))$$

So μ is shorthand for the content of Rosen's definition of definition. Nothing substantive turns on this shorthand; the problem could be straightforwardly articulated without it. Nevertheless, it would be technically unwieldy, so I help myself to the abbreviation. Rosen's definition of definition thus becomes:

$$\text{Def}(F, \phi) \text{ iff } \mu \tag{1}$$

Property F is, by definition, ϕ just in case μ. When his account is applied to itself (i.e., when definition is itself the relation subject to definition), we obtain:

$$\text{Def}(\text{Def}, \mu) \text{ iff } \Box_{\text{Def}}\forall_{<\text{F}, \phi>}((\text{Def}(F, \phi) \vee \mu) \rightarrow (\text{Def}(F, \phi) \leftarrow \mu)) \tag{2}$$

This is simply the result of 'plugging in' the definition relation to Rosen's own proposed account of definition. If Rosen has indeed provided a definition of definition, self application is surely required. This asserts that definition is defined in terms of μ just in case it is necessary in virtue of the nature of definition that, for all ordered pairs $<F, \phi>$, if either F is by definition ϕ or μ, then the fact that F is by definition ϕ is grounded in the fact that μ. The substitution principle and equation (1) license the substitution of occurrences of 'Def(F, ϕ)' with μ in the definiens of equation (2). One such application results in:

$$\text{Def}(\text{Def}, \mu) \text{ iff } \Box_{\text{Def}}\forall_{<\text{F}, \phi>}((\text{Def}(F, \phi) \vee \mu) \rightarrow (\mu \leftarrow \mu)) \tag{3}$$

Equation 3 contains a reflexive grounding relation: $\mu \leftarrow \mu$. Given that (I take it) Rosen's definition of definition is intended to be true, it follows that there is at least one reflexive grounding relation: the fact that μ grounds the fact that μ. This contradicts the assumption that there are no reflexive grounding relations.

A contradiction thus arose from assumptions a), b) and c). Given that Rosen is committed to all these claims, his commitments are logically inconsistent. Of course, he could retain his commitment to definitional substitution by allowing for reflexive grounding relations (or, alternately, by abandoning his account of definition, but I doubt he would find this alternative appealing).

The reflexivity of grounding may be troubling enough, but reflexive definitions are presently at issue. Employing the substitution principle twice more yields:

$$\text{Def}(\text{Def}, \mu) \text{ iff } \Box_{\mu}\forall_{<\text{F}, \phi>}((\mu \vee \mu) \rightarrow (\mu \leftarrow \mu)) \tag{4}$$

It is necessary in virtue of the identity of μ that, for all ordered pairs $<F, \phi>$, if either μ or μ, then the fact that μ is grounded in the fact that μ. This, of course, is what is required for μ to be defined in terms of itself on Rosen's account, i.e.,

$$\text{Def}(\mu, \mu) \text{ iff } \Box_{\mu} \forall_{<F, \phi>}((\mu \vee \mu) \rightarrow (\mu \leftarrow \mu)) \tag{5}$$

And, from 4, 5 and the transitivity of the classical biconditional, we obtain:

$$\text{Def}(\text{Def}, \mu) \text{ iff } \text{Def}(\mu, \mu) \tag{6}$$

So definition is defined in terms of μ just in case μ is defined directly and exclusively in terms of itself.[4] The restricted substitution principle that Rosen employs thus gives rise to reflexive definitions.

4 Definition as an Identity

Some may suspect that reflexivity was peculiar to Rosen's account. After all, metaphysicians have objected on independent grounds.[5] Perhaps a more plausible account of definition could retain substitution principles while avoiding definitional reflexivity.

To that end, let us set Rosen's account aside and consider a prominent alternative. It has been suggested that definitions are a subset of identities. Correia (2017), for example, argues that real definitions are sentences of the form 'To be *F* is to be *G*' where the 'is' is the 'is' of identity, and where the fact that something is *G* grounds the fact that it is *F*.[6] So, for example, 'To be a brother is to be a male sibling' expresses a definition just in case the property of *being a brother* is identical to the property of *being a male sibling* and *being a male sibling* grounds *being a brother*.

There are important details of Correia's account that warrant further discussion—in particular, he defends a generic, non-factive notion of grounding and strictly maintains that definitions merely *strongly resemble* identity claims without the ontological commitment to properties that identity claims have. However, nothing in the present discussion turns on these complications, so I set them aside.

On this conception, sentences that express definitions have three components: a term for the definiendum, a term for the definiens, and an identity sign linking the two. In English, the definiendum typically appears to the left of the identity sign, while the definiens typically appears to the right. If 'to be human is to be the rational animal'

[4]Rosen is also committed to the claim that there are no reflexive definitions—see pg. 201. That his account entails the existence of reflexive definitions is another, yet related, point of inconsistency.

[5]See, e.g., Correia (2017).

[6]Or, alternately, the fact that something is *G* is more natural than the fact that it is *F*, in Lewis's sense of 'natural.'

expresses a definition, 'to be human' is the term for the definiendum, while 'to be the rational animal' is the term for the definiens. The examples I discuss here distinguish the definiendum from the definiens by this ordering.

Let us say that a sentence *expresses a reflexive definition* just in case it both expresses a definition and the terms for the definiendum and definiens are identical—that is to say, if and only if the sentence takes the form $\ulcorner \phi =_{def} \phi \urcorner$. If 'to be morally right is to be morally right' expresses a definition, then it expresses a reflexive definition. A constraint against definitional reflexivity, then, amounts to the claim that no sentences express reflexive definitions.

5 From Substitution to Reflexivity (Part 2)

A substitution principle on this conception of definition looks much like the one advanced by Rosen:

$$\begin{array}{c} 7.\ \phi =_{Def} \psi \\ 8.\ \varphi =_{Def} \gamma \\ \hline 9.\ \phi =_{Def} \psi^{\varphi/\gamma} \end{array}$$

If ϕ is, by definition ψ and φ is, by definition, γ, then replacing occurrences of φ with γ in ψ results in a definition of ϕ. This principle has intuitive applications. If 'To be {Socrates} is to be the set containing only Socrates' and 'Socrates is the result of this sperm and that egg' both express definitions, then 'To be {Socrates} is to be the set containing only the result of this sperm and that egg' also expresses a definition.

Let us temporarily suppose that propositions (non-linguistic bearers of truth-values) are defined in terms of their logical forms.[7] The proposition that grass is green and the sky is blue is—by definition—a conjunction. In particular, it is the conjunction of the proposition that grass is green with the proposition that the sky is blue. The proposition that it is not the case that the Sun revolves around the Earth is—by definition—a negation. In particular, it is the negation of the proposition that the Sun revolves around the Earth. Generally:

$$p \wedge q =_{Def} \wedge(p, q)$$
$$p \vee q =_{Def} \vee(p, q)$$
$$\neg p =_{Def} \neg(p)$$

[7] For an account of propositions along these lines, see Bealer (1998).

Further, let us assume that there is a proposition that asserts that $p \wedge q$ is defined in terms of its logical form. Let us denote this proposition as $[p \wedge q =_{def} \wedge(p,q)]$, and consider the conjunction of *that* proposition with an arbitrary proposition r. If propositions are defined in terms of their logical form, then this conjunction is itself defined in terms of its logical form, i.e.:

$$p \wedge q =_{Def} \wedge (p,q) \tag{7}$$

$$r \wedge [p \wedge q =_{Def} \wedge (p,q)] =_{Def} \wedge (r, [p \wedge q =_{Def} \wedge (p,q)]) \tag{8}$$

Both (7) and (8) obtain if propositions are defined in terms of their logical form. The substitution principle and 7 license the substitution of $p \wedge q$ with $\wedge(p,q)$ in the definiens of (8). This results in:

$$r \wedge [p \wedge q =_{Def} \wedge (p,q)] =_{Def} \wedge (r, [\wedge(p,q) =_{Def} \wedge (p,q)]) \tag{9}$$

Note that the definiendum of (9) contains a reflexive definition: $\wedge(p,q) =_{Def} \wedge (p,q)$. If we let r be any true proposition, then the *definiendum* of equation (9) is a proposition which is true. After all, it is the conjunction of two true propositions. Given that, on this conception of definition, the definiens is identical to the definiendum, it follows that the *definiens* in (9) is also a proposition which is true.[8]. But the definiens is the result of conjoining r with a proposition that asserts a reflexive definition. In order for this conjunction to be true, the following must be true as well:

$$\wedge(p,q) =_{Def} \wedge (p,q) \tag{10}$$

This takes the form $\ulcorner \phi =_{def} \phi \urcorner$. Therefore, there are reflexive definitions.

If we drop the assumption that propositions are defined in terms of their logical form then this particular example may fail. However, it is an instance of a general pattern that remains. The key is to build one definition within the definiens of another. For, armed with the following two sentences:

$$A =_{\text{Def}} (...B =_{Def} C...) \tag{11}$$

$$B =_{\text{Def}} C \tag{12}$$

[8] I assume that, in order for two positions to be identical, they must share the same truth-value.

The substitution principle licenses

$$A =_{Def} (...C =_{Def} C...) \tag{13}$$

So long as this type of construction is possible, the substitution principle engenders reflexive definitions.

6 Conclusion

On two standard conceptions of real definition, definitional substitution gives rise to reflexive definitions. This holds even if the substitution principle is restricted to the definiens of a definition. This has several philosophically significant implications that bear mentioning at this time. The first concerns the opacity of definition. Some relations are said to be *transparent*, while others are said to be *opaque*. The distinction between the two is frequently understood in terms of the validity of substitution principles. If co-referential terms can be substituted for one another in a relation *salva veritate*, the relation is said to be transparent. If they cannot be so substituted, the relation is said to be opaque.

Most standard relations are transparent; the relation *being 6 feet tall* is an unremarkable example. If 'John is 6 feet tall' is true, then the substitution of 'John' with another term that denotes the same person results in a true sentence. The term 'believes' is the most canonical opaque relation. It may be that 'Lois Lane believes that Clark Kent is a reporter' is true, while 'Lois Lane believes that Superman is a reporter' is false, despite the fact that 'Clark Kent' and 'Superman' denote the same person.

If substitution principles for definition fail, then definition is an opaque relation. The term 'definition' functions like 'believes' rather than 'is 6 feet tall.' This is not the first argument that definition is opaque (see Correia (2017)), but it is a new argument to that effect.

A second implication is methodological; unless philosophers countenance reflexive definitions, they cannot infer from the fact that entity A is defined in terms of entity B, and entity B is defined in terms of entity C that entity A is defined in terms of entity C. So, for example, if philosophers were to argue that sets are defined in terms of their members and that people are defined in terms of their genetic makeup, they ought not infer that {Socrates} is defined in terms of Socrates' genetic makeup.

A third implication is metaphysical. Fine (1995) has argued that ontological dependence ought to be understood in terms of real definition. Somewhat loosely, A ontologically depends upon B just in case a term that denotes B is a component of the definiens in a sentence that expresses a definition for which A is the definiendum. On this conception, the logical features of ontological dependence rely on the logical features of definition. If definition does not admit of substitution, then ontological dependence is intransitive. A

may ontologically depend upon B, and B may ontologically depend upon C despite the fact that A does not ontologically depend upon C.

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