Does Definition Admit of Substitution?

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Abstract

I argue that definitional substitution engenders reflexive definitions: cases in which an entity is defined directly and exclusively in terms of itself. I consider two prominent accounts of definition—one according to which definition is a relation between properties and structured complexes, and the other according to which definition is a species of identity—and demonstrate that substitution principles lead to reflexive definitions for each. Along the way, I demonstrate that the claims in 'Real Definition' (Rosen (2015)) are logically inconsistent. I close with a brief discussions of the implications this has for the opacity of definition and for philosophical methodology more generally.

1 Introduction

Real definition plays an integral role not only in metaphysics, but in nearly all branches of philosophy.[1] Some moral disputes can be understood as a disagreement over the definition of the good; attempted amendments to a justified-true-belief analysis of knowledge as a search for the definition of knowledge; and one type of physicalism as the claim that everything is defined in purely physical terms. Much philosophy is concerned with the identity of things—with what it is that makes entities the entities that they are. The nature of definition is important, at least in part, because it directly impacts much of philosophical inquiry.

This paper concerns a logical feature of definition—with what follows from definitional claims. A bit loosely, I am concerned with whether definitions can be *substituted* within one another. Some may suspect that definition admits of substitution. If water is defined in terms of its atomic constituents, and its atomic constituents are defined in terms of their subatomic parts, it seems reasonable to maintain that water is ultimately defined in terms of its subatomic parts. Within the definition of water, the thought goes, water's atomic

[1] By 'real definition' I mean the metaphysical analogue of nominal definition. While the nominal definition of 'water' is a definition of the term, the real definition of water is a definition of the compound itself. While some dispute the distinction between real and nominal definition, for the purposes of this paper I will assume that it is coherent. Here, I use 'definition' to refer to real definition unless otherwise specified.

constituents can be replaced by their own definition. This presumably has nothing to do with water in particular. Rather, it is a logical feature of definition; any time one entity is defined in terms of another, the latter can be replaced with its own definition within the definition of the former. So, if the property of *being a bachelor* is—by definition—the property of *being an unmarried male* and the property of *being unmarried* is—by definition—the property of *lacking a marriage*, then the property of *being a bachelor* is—by definition—the property of *being a male who lacks a marriage*. And if {2} is—by definition—the set containing only the number 2 and the number 2 is—by definition—the successor of the number 1, then {2} is—by definition—the set containing only the successor of the number 1.

I maintain that this is false; real definition does not admit of substitution. I do not claim that there are no cases in which definitional substitution succeeds. Rather, I claim that even modest substitution principles entail that there are reflexive definitions—cases in which an entity is defined directly and exclusively in terms of itself. I reject reflexive definitions, and suspect that I am in good company in doing so. Strange as the literature on personal identity may be, it has never been suggested that Socrates's definition is Socrates himself. And while some argue that knowledge is primitive, no one has suggested that knowledge is—by definition—knowledge. However, I have no new argument against reflexive definitions at present, so all that I take myself to demonstrate is a conditional: *if* definition admits of substitution, *then* there are reflexive definitions (or, alternatively, that *either* definition does not admit of substitution, *or* there are reflexive definitions). I am personally more averse to reflexive definitions than wed to substitution principles, so I take this to constitute a reason to abandon substitution principles. Those who do not mind reflexive definitions, however, may retain substitution principles.

On one interpretation of 'substitution,' this conclusion may be fairly unsurprising. Suppose that we license the following substitution principle (without yet taking a stand on what definition itself consists of):

$$\frac{\text{i) } A \text{ is—by definition—} B \qquad \text{ii) } C \text{ is—by definition—} D}{\text{iii) } A \text{ is—by definition—} B^{C/D}}$$

That is to say, if A is—by definition—B, and C is—by definition—D, then any replacement of C with D within 'A is—by definition—B' is valid. It is easy to see that this accommodates the examples mentioned above. For instance, if we allow sentence i) to be '{2} is—by definition—the set containing only the number 2' and allow sentence ii) to be 'the number 2 is—by definition—the successor of the number 1,' this principle licenses '{2} is—by definition—the set containing only the successor of the number 1.'

This principle quickly generates reflexive definitions. In fact, it generates reflexive definitions so quickly that I have never seen anyone actually endorse it. Suppose, for example, that to be a vixen is—by definition—to be a female fox. If we allow 'To be a vixen is—by definition—to be a female fox' to be both sentence i) and sentence ii), then the substitution principle licenses 'To be a female fox is—by definition—to be a female fox.' Of course, this is not peculiar to female foxes. Quite generally, allowing sentences i) and ii) to be the same generates reflexive definitions.

Presumably for this reason, philosophers have preferred more restricted substitution principles.[2] Reflexivity arose, some might reasonably suspect, because the principle licensed substitution within the definiendum—or the object of analysis. If substitution principles were restricted to the definiens—or the content of analysis, then reflexivity would be avoided.

What I take to be surprising is that these more restricted substitution principles *also* give rise to reflexive definitions. Here, I explore two dominant conceptions of definition. The first, endorsed by Rosen (2015), is that real definition is a relation between a property and a structured complex. For example, it may be a relation that holds between the property *being a father* and the complex of being a male parent. The second, endorsed by Correia (2017), is that real definitions are a kind of identity.[3] For example, it may be that 'To be even is to be a natural number divisible by 2 without remainder' expresses a definition. On each conception, restricted substitution principles give rise to reflexive definitions, so the link between substitution and reflexivity does not turn on which account of definition we adopt.

I do not canvass everything that might reasonably count as a substitution principle. Perhaps some do not engender reflexive definitions. Nevertheless, I take it that the principles I address are reasonably called 'substitution principles,' and, I suspect, are what many philosophers have in mind when discussing definitional substitution.

I proceed as follows. In section 2, I present the view that definition is a relation between properties and structured complexes. In section 3, I demonstrate that definitional substitution generates reflexive definitions on this view. As a corollary, I demonstrate that three central claims in Rosen (2015) are logically inconsistent. In section 4, I present the view that definition is a species of identity, and show how substitution principles generate reflexive definitions on this view in section 5. In section 6, I argue that the link between substitution and reflexivity are not peculiar to these conceptions of definition, and I conclude in section 7 by briefly discussing the implications this has for the opacity of definition and for philosophical methodology.

[2]For those who endorse substitution in this area, see, e.g., Rosen (2015); Dorr (2016); Correia and Skiles (Forthcoming). Additionally, Fine (2015); Horvath (2017) endorse the transitivity of definition, which closely resembles substitution principles.

[3]More precisely, definitions are a species of sentences that closely resemble identities without the ontological commitment that the terms flanking the identity sign denote. For a more precise discussion of Correia's proposal, see section 4.

2 Definition as a Relation

On one conception, definition is a relation. In particular, Rosen maintains that definition is a relation that obtains between properties (and relations) and structured complexes. Rosen provides no precise account of what a structured complex consists of, saying that it is "built from worldly items in roughly the sense in which a sentence is built from words" (Rosen, 2015, pg. 190). On this view, structured complexes closely resemble Russellian propositions except for the presence of free variables that correspond to the adicity of the property or relation being defined. For example, if being a person is—by definition—being a rational animal, then the definition relation obtains between the property *being a person* and the complex *being a rational animal*, and if being a hydrogen atom is—by definition—being an atom whose nucleus contains exactly one proton, then the definition relation obtains between the property *being a hydrogen atom* and the structured complex *being an atom whose nucleus contains exactly one proton*.

Problems may already arise. If structured complexes so closely resemble structured propositions, they might inherit obstacles that structured propositions face. For example, Goodman (2017) recently argued that Russell's paradox threatens many conceptions of structured propositions.[4] It may be difficult to characterize the notion of a structured complex in a logically consistent way. For the moment, let us set this type of worry aside.

Notably, Rosen's account does not require that definitions are identities. Even if *being a mother* is defined in terms of the complex of being a female parent, the two may be distinct from one another. If the relevant structured complexes are not themselves properties or relations, an application of Leibniz's law ensures that a definiendum is distinct from its definiens; after all, the definiendum bears the property *being a property or relation*, while the definiens does not.

Rosen's account employs two notational elements that surpass traditional second-order logic. The first is the indexed modal operator $\Box_F$, which appeared first in Fine (1995). Roughly, this can be translated to 'it is necessary in virtue of the identity of F,' so '$\Box_{Knowledge}$(Knowledge is a mental state)' means that it is necessary in virtue of the identity of knowledge that knowledge is a mental state, and '$\Box_{\text{Better-than}}$(Better-than is transitive)' means that it is necessary in virtue of the identity of the relation *being better than* that better-than is transitive.

The second symbol denotes *grounding*—a relation of metaphysical dependence. Grounding is sometimes described as a non-causal explanatory relation, and is often expressed with phrases like 'because' or 'in-virtue-of.' It may be that the fact that a ball is both red and round is grounded in (or holds because, or obtains in-virtue-of) the fact that it is red and the fact that it is round, and it may be that the fact that a particular substance is water is grounded in the fact that the substance is the chemical compound H_2O.

Rosen takes grounding to be a many-one relation that obtains between facts, and

[4] To his credit, this is a worry that Rosen foresees.

symbolizes it with the arrow '$\leftarrow$,' so '$A \vee B \leftarrow A$' means that the fact that A or B is grounded in the fact that A. Additionally, Rosen makes the standard (but not uncontroversial) assumption that ground is a strict partial ordering, i.e., that the following conditions on grounding obtain:

> 1. Irreflexivity: $\neg(A \leftarrow A)$ (no fact grounds itself).
> 2. Asymmetry: $(A \leftarrow B) \rightarrow \neg(B \leftarrow A)$ (if one fact grounds a second, then the second does not ground the first).
> 3. Transitivity: $(A \leftarrow B) \wedge (B \leftarrow C) \rightarrow (A \leftarrow C)$ (if one fact grounds a second,
> and the second grounds a third, then the first grounds the third. (pg. 201)

Armed with these notions, Rosen provides the following account of definition:

$$Def(F, \phi) \text{ iff } \Box_{\mathrm{F}} \forall x((Fx \vee \phi x) \rightarrow (Fx \leftarrow \phi x))$$

This states that a property F is—by definition—the structured complex ϕ just in case it is necessary in virtue of the identity of F that, for any object, if that object is either F or ϕ, then it is F in virtue of being ϕ (i.e., the fact that the object is F is grounded in the fact that it is ϕ). For example, the property of *being morally right* is defined in terms of maximizing utility just in case it is necessary in virtue of the identity of *being morally right* that if any act is either morally right or maximizes utility, then the fact that it maximizes utility grounds the fact that it is morally right.

Although Rosen states his account biconditionally, he intends something stronger. The definition relation itself is a candidate relation subject to definition. Rather than merely providing necessary and sufficient conditions for real definition, Rosen takes himself to have provided a *definition* of definition.

3 From Substitution to Reflexivity (Part 1)

Rosen licenses definitional substitution remarkably explicitly, asserting:

> It should be possible to prove a principle that licenses arbitrary definitional expansion: Def(F, ϕ) and Def(G, ψ) then Def(F, $\phi^{\psi/G}$) where $\phi^{\psi/G}$ is the result of substituting ψ for G in ϕ...Any account of real definition should license the substitutions of definiens for definiendum in a ground to yield a further ground. (Rosen, 2015, pg. 201)

Definitional substitution is not only an inference Rosen endorses, but ought to be *provable* on an adequate account of definition. He restricts definitional substitution to the definiens; it is admissible only the content of analysis, not the object of analysis. In particular, he licenses the following substitution principle:

$$\begin{array}{c}\text{iv) Def}(F,\, \phi) \\ \text{v) Def}(G,\, \psi) \\ \hline \text{vi) Def}(F,\, \phi^{\psi/G})\end{array}$$

I disagree with Rosen. I maintain that this principle is invalid. To wit, this principle contradicts other claims Rosen makes. In particular, the following commitments of Rosen's are logically inconsistent:

a) Grounding is irreflexive
b) The substitution principle
c) Rosen's definition of definition

The inconsistency emerges when Rosen's account is applied to itself: when the definition relation is itself subject to definition. Demonstrating this quickly becomes technically cumbersome, so I help myself to the following abbreviation:

$$\mu =_{def} \Box_{\mathrm{F}}\, \forall x((Fx \vee \phi x) \rightarrow (Fx \leftarrow \phi x))$$

So μ is shorthand for the content of Rosen's definition of definition. Nothing substantive turns on this shorthand; the problem could be straightforwardly articulated without it. Nevertheless, it would be technically unwieldy, so I help myself to the abbreviation. Rosen's definition of definition thus becomes:

$$\text{Def}(F, \phi) \text{ iff } \mu \tag{1}$$

Property F is—by definition—ϕ just in case μ. When his account is applied to itself (i.e., when definition is itself the relation subject to definition), we obtain:

$$\text{Def}(\text{Def}, \mu) \text{ iff } \Box_{\text{Def}} \forall_{<\text{F}, \phi>}((\text{Def}(F, \phi) \vee \mu) \rightarrow (\text{Def}(F, \phi) \leftarrow \mu)) \tag{2}$$

This is simply the result of 'plugging in' the definition relation to Rosen's own proposed account of definition. If Rosen has indeed provided a definition of definition, self-application is surely required. This asserts that definition is defined in terms of μ just in case it is necessary in virtue of the nature of definition that, for all ordered pairs $<F, \phi>$, if either F is by definition ϕ or μ, then the fact that F is by definition ϕ is grounded in the fact that μ. The substitution principle and equation (1) license the substitution of occurrences of 'Def(F, ϕ)' with μ in the definiens of equation (2). One such application results in:

$$\text{Def}(\text{Def}, \mu) \text{ iff } \Box_{\text{Def}} \forall_{<\text{F}, \phi>}((\text{Def}(F, \phi) \vee \mu) \rightarrow (\mu \leftarrow \mu)) \tag{3}$$

Equation 3 contains a reflexive grounding relation: $\mu \leftarrow \mu$. Given that (I take it) Rosen's definition of definition is intended to be true, it follows that there is at least one reflexive grounding relation: the fact that μ grounds the fact that μ. This contradicts the assumption that there are no reflexive grounding relations.

A contradiction thus arose from assumptions a), b) and c). Given that Rosen is committed to all these claims, his commitments are logically inconsistent. Of course, he could retain his commitment to definitional substitution by allowing for reflexive grounding relations (or, alternately, by abandoning his account of definition, but I doubt he would find this alternative appealing).

The reflexivity of grounding may be troubling enough, but reflexive definitions are presently at issue. Employing the substitution principle twice more yields:

$$\text{Def}(\text{Def}, \mu) \text{ iff } \Box_{\mu} \forall_{<F, \phi>}((\mu \vee \mu) \rightarrow (\mu \leftarrow \mu)) \tag{4}$$

It is necessary in virtue of the identity of μ that, for all ordered pairs $<F, \phi>$, if either μ or μ, then the fact that μ is grounded in the fact that μ. This, of course, is what is required for μ to be defined in terms of itself on Rosen's account, i.e.,

$$\text{Def}(\mu, \mu) \text{ iff } \Box_{\mu} \forall_{<F, \phi>}((\mu \vee \mu) \rightarrow (\mu \leftarrow \mu)) \tag{5}$$

And, from 4, 5 and the transitivity of the classical biconditional, we obtain:

$$\text{Def}(\text{Def}, \mu) \text{ iff } \text{Def}(\mu, \mu) \tag{6}$$

So definition is defined in terms of μ just in case μ is defined directly and exclusively in terms of itself.[5] The restricted substitution principle that Rosen employs thus gives rise to reflexive definitions.

4 Definition as an Identity

Some may suspect that reflexivity was peculiar to Rosen's account. After all, metaphysicians have objected to it for independent reasons. In addition to the worries about structured complexes mentioned before, Correia (2017) objects on the grounds that an account of real definition ought to guarantee that a definiendum is identical to its definiens. Perhaps a more plausible account of definition could retain substitution principles while avoiding definitional reflexivity.

[5] Rosen is also committed to the claim that there are no reflexive definitions—see pg. 201. That his account entails the existence of reflexive definitions is another, yet related, point of inconsistency.

To that end, let us set Rosen's account aside and consider a prominent alternative. Recently, there has been substantial interest in a kind of sentence dubbed 'generalized identities.' These are sentences of the form 'To be *F* is to be *G*', where the 'is' strongly resembles the 'is' of identity.[6] Notable examples include 'To be a bachelor is to be an unmarried male' and 'To be just is to be such that each part of one's soul does its own proper work.' Given that the 'is' of these sentences so closely resembles the 'is' of identity, it is tempting to suggest that generalized identities simply state the identities of properties and relations; 'To be *F* is to be *G*' is true just in case the property of *being F* is identical to the property of *being G*. However, several philosophers object to this interpretation.[7] Minimally, there is linguistic evidence that 'To be *F*' is not synonymous with 'To be the property of *being F*' in at least some contexts. The sentence 'I hope to be an accomplished philosopher' is perfectly true, yet 'I hope to be the property of being an accomplished philosopher' is presumably false—I do not hope to be a property. In addition, some desire accounts of generalized identities to be compatible with nominalism: the contention that abstract objects like properties and relations do not exist. If 'To be *F* is to be *G*' were synonymous with 'To be the property of being *F* is identical to be the property of being *G*,' nominalists would lack the resources to avail themselves of generalized identities from the outset. For these reasons, some argue that generalized identities strongly resemble, but are not strictly, identity claims.

Notwithstanding these refinements, generalized identities are standardly taken to share the logical and modal profile of identity. In particular, they are transitive, reflexive and symmetric, and if 'To be *F* is to be *G*' is true then it is necessarily true, and necessary that all and only *F*s are *G*s. Given that generalized identities are symmetric, it appears that definitions (which are often taken to be asymmetric) cannot immediately be identified with the generalized identities. If to be a square is—by definition—to be an equilateral rectangle, then to be an equilateral rectangle is not—by definition—to be a square.

Correia (2017) argues that definitions are kind of generalized identity: the set of definitions is a subset of the set of generalized identities. They are generalized identities that satisfy a further requirement. Correia proposes winnowing down the generalized identities into those that are definitions two ways—one which employs the notion of grounding and the other which employs Lewis's notion of relative naturalness. The relation between these potential refinements is important in its own right, but does not impact the issue of substitution. For the purpose of this paper, I will restrict my attention to his characterization in terms of grounding. Correia's notion of ground differs slightly from Rosen's. In particular, he does not operate with a factive notion of ground—the kinds of things that can stand in grounding relates generics (like *being F* and *being G*) rather than particular facts.[8] With this notion of ground in hand, Correia advances the following account:

[6]See, e.g., Dorr (2016); Rayo (2013); Linnebo (2014).
[7]See Dorr (2016).
[8]More precisely, they are *representations* of generics, but this further refinement is not needed here.

To be F is$_{df}$ to be G if and only if:

1) To be F is$_{id}$ to be G.[9]
2) Being G grounds being F.

Let us say that, on this account, a sentence *expresses a reflexive definition* just in case it both expresses a definition and the terms for F and G are identical, i.e., just in case there is an instance of 'To be F is to be F' that satisfies both conditions 1 and 2. If 'to be morally right is to be morally right' expresses a definition, then it expresses a reflexive definition. A constraint against definitional reflexivity, then, amounts to the claim that no sentences express reflexive definitions.

5 From Substitution to Reflexivity (Part 2)

Correia does not endorse definitional substitution as explicitly as Rosen does. However, he does maintain that definition is transitive because both conditions 1 and 2 are transitive. In a related paper, Correia and Skiles (Forthcoming) come closer and employ a substitution principle according to which if to be H is a part of what it is to be F, and to be F is to be G, then to be H is a part of what it is to be G (pg. 19). So, for example, if being rational is a part of what it is to be a rational animal, and to be human is to be a rational animal, then being rational is a part of what it is to be human.

A precise substitution principle on this conception of definition looks very similar to Rosen's:

$$\frac{\text{vii) To be } F \text{ is}_{df} \text{ to be } G \qquad \text{viii) To be } H \text{ is}_{df} \text{ to be } I}{\text{ix) To be } F \text{ is}_{df} \text{ to be } G^{\mathrm{H/I}}}$$

If to be F is—by definition—to be G and if to be H is—by definition—to be I, then replacing occurrences of H with I within G results in a definition of F. So, for example, if 'To be {Socrates} is to be the set containing only Socrates' and 'To be Socrates is to be the result of this sperm and that egg' both express definitions, then 'To be {Socrates} is to be the set containing only the result of this sperm and that egg' also expresses a definition.

As before, I help myself to an abbreviation to alleviate excessive formalisms:

$$\omega =_{def} \text{to be } F \text{ is}_{id} \text{ to be } G \wedge \text{Being } G \text{ grounds being } F$$

[9]Following Correia, I distinguish the reading of 'To be F is to be G' that resembles an identity from the reading that resembles a definition with the subscripts '*id*' and '*df*'. 'To be F is$_{id}$ to be G' is to be read nearly synonymously with 'Being F is identical to being G,' while 'To be F is$_{df}$ to be G' is to be read as 'To be F is—by definition—to be G.'

ω is shorthand for the conjunction of the content of Correia's account. Like Rosen, Correia takes himself to be provided a definition of definition; the definition relation itself is one of the things which could be defined. His proposal thus becomes:

$$\text{To be definition is}_{df} \text{ to be } \omega \tag{7}$$

Given the specifics of his account, this requires:

$$\text{To be definition is}_{df} \text{ to be } \omega \text{ iff to be definition is}_{id} \text{ to be } \omega \wedge \text{ being } \omega \text{ grounds being definition.} \tag{8}$$

The substitution principle and (7) license substituting occurrences of 'definition' with ω in the definiens of sentence (8). An application of this results in:

$$\text{To be definition is}_{df} \text{ to be } \omega \text{ iff to be definition is}_{id} \text{ to be } \omega \wedge \text{ being } \omega \text{ grounds being } \omega. \tag{9}$$

As with Rosen's account, Correia's proposal generates a reflexive grounding relation under the assumption definition is substitutable—being ω grounds being ω. Another application of the substitution principle results in:[10]

$$\text{To be definition is}_{df} \text{ to be } \omega \text{ iff to be } \omega \text{ is}_{id} \text{ to be } \omega \wedge \text{ being } \omega \text{ grounds being } \omega. \tag{10}$$

This is what is required for ω to be defined in terms of itself, i.e.,

$$\text{To be } \omega \text{ is}_{df} \text{ to be } \omega \text{ iff to be } \omega \text{ is}_{id} \text{ to be } \omega \wedge \text{ being } \omega \text{ grounds being } \omega. \tag{11}$$

From 10, 11 and the transitivity of the classical biconditional, we obtain:

$$\text{To be definition is}_{df} \text{ to be } \omega \text{ iff to be } \omega \text{ is}_{df} \text{ to be } \omega. \tag{12}$$

Definition is defined in terms of ω just in case ω is defined directly and exclusively in terms of itself. The substitution principle thus gives ruse to reflexive definition on Correia's account. Of course, Correia may not support the substitution principle that I put forward—this is not a hit piece. But those who accept Correia's account would do well to disavow substitution principles, because they give rise to reflexive definitions.

[10] Although the substitution principle licenses this, it is not strictly necessary for the derivation of (10) from (9). Given that generalized identities are reflexive, we already had that to be ω is_{id} to be ω, which is the only difference between (9) and (10). However, the principle was needed to derive (9) from (8).

6 Substitution and Reflexivity Generally

The derivation of reflexive definitions was similar enough on Rosen's and Correia's accounts that, plausibly, it is unrelated to their particular proposals. Perhaps any definition of definition has a similar result. Select an arbitrary account of definition—one according to which the definition relation itself is defined. Let us denote the content of an account, whatever it may be, with τ. Because it is a definition of definition, self-application is required—definition itself is subject to definition. However, the substitution principle licenses replacing occurrences of definition with occurrences of τ in the definiens of this self-application. The relation that definition stands in to τ thus becomes the relation that τ stands in to itself. So, if definition is defined in terms of τ, then τ is defined in terms of τ, and there are reflexive definitions.

I note, however, that reflexivity is not only generated in this type of case. Generating others, however, usually depends on controversial philosophical commitments. Let us suppose that propositions (non-linguistic bearers of truth-values) are defined in terms of their logical forms.[11] The proposition that grass is green and the sky is blue is—by definition—a conjunction. In particular, it is the conjunction of the proposition that grass is green with the proposition that the sky is blue. The proposition that it is not the case that the Sun revolves around the Earth is—by definition—a negation. In particular, it is the negation of the proposition that the Sun revolves around the Earth. Generally:

$$p \wedge q =_{Def} \wedge(p, q)$$
$$p \vee q =_{Def} \vee(p, q)$$
$$\neg p =_{Def} \neg(p)$$

Further, let us assume that there is a proposition that asserts that $p \wedge q$ is defined in terms of its logical form. Let us denote this proposition as $[p \wedge q =_{def} \wedge(p, q)]$, and consider the conjunction of *that* proposition with an arbitrary proposition r. If propositions are defined in terms of their logical form, then this conjunction is itself defined in terms of its logical form, i.e.:

$$p \wedge q =_{Def} \wedge (p, q) \tag{13}$$

$$r \wedge [p \wedge q =_{Def} \wedge (p, q)] =_{Def} \wedge (r, [p \wedge q =_{Def} \wedge (p, q)]) \tag{14}$$

[11] For an account of propositions along these lines, see Bealer (1998).

Both (13) and (14) obtain if propositions are defined in terms of their logical form. The substitution principle and (13) license the substitution of $p \wedge q$ with $\wedge(p, q)$ in the definiens of (14). This results in:

$$r \wedge [p \wedge q =_{Def} \wedge (p, q)] =_{Def} \wedge (r, [\wedge(p, q) =_{Def} \wedge (p, q)]) \tag{15}$$

Note that the definiens of (15) contains a reflexive definition: $\wedge(p, q) =_{Def} \wedge(p, q)$. If we let r be any true proposition, then the *definiendum* of equation (15) is a proposition which is true. After all, it is the conjunction of two true propositions. Given that, on this conception of definition, the definiens is identical to the definiendum, it follows that the *definiens* in (15) is also a proposition which is true.[12] But the definiens is the result of conjoining r with a proposition that asserts a reflexive definition. In order for this conjunction to be true, the following must be true as well:

$$\wedge(p, q) =_{Def} \wedge (p, q) \tag{16}$$

Because $\wedge(p, q)$ is defined directly and exclusively in terms of itself, there are reflexive definitions.

7 Conclusion

On two standard conceptions of real definition, definitional substitution gives rise to reflexive definitions. This holds even if the substitution principle is restricted to the definiens of a definition. This has several philosophically significant implications that bear mentioning at this time. The first concerns the opacity of definition. Some relations are said to be *transparent*, while others are said to be *opaque*. The distinction between the two is frequently understood in terms of the validity of substitution principles. If co-referential terms can be substituted for one another in a relation *salva veritate*, the relation is said to be transparent. If they cannot be so substituted, the relation is said to be opaque.

Most standard relations are transparent; the relation *being 6 feet tall* is an unremarkable example. If 'John is 6 feet tall' is true, then the substitution of 'John' with another term that denotes the same person results in a true sentence. The term 'believes' is the most canonical term for an opaque relation. It may be that 'Lois Lane believes that Clark Kent is a reporter' is true, while 'Lois Lane believes that Superman is a reporter' is false, despite the fact that 'Clark Kent' and 'Superman' denote the same person.

If substitution principles for definition fail, then definition is an opaque relation. The term 'definition' functions like 'believes' rather than 'is 6 feet tall.' This is not the first

[12] I assume that, in order for two positions to be identical, they must share the same truth-value.

argument that definition is opaque (see Correia (2017)), but it is a new argument to the same effect.

A second implication is methodological; unless philosophers countenance reflexive definitions, they cannot infer from the fact that entity *A* is defined in terms of entity *B*, and entity *B* is defined in terms of entity *C* that entity *A* is defined in terms of entity *C*. So, for example, if philosophers were to argue that sets are defined in terms of their members and that people are defined in terms of their genetic makeup, they ought not infer that {Socrates} is defined in terms of Socrates' genetic makeup.

A third implication is metaphysical. Fine (1995) has argued that ontological dependence ought to be understood in terms of real definition. Somewhat loosely, *A* ontologically depends upon *B* just in case a term that denotes *B* is a component of the definiens in a sentence that expresses a definition for which *A* is the definiendum. On this conception, the logical features of ontological dependence rely on the logical features of definition. If definition does not admit of substitution, then ontological dependence is intransitive. *A* may ontologically depend upon *B*, and *B* may ontologically depend upon *C* despite the fact that *A* does not ontologically depend upon *C*.

References

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