Propositional Logic Review I

1 Definitions and Terminology

Premises: An argument's *premises* are the claims that you assume (or suppose) to be true at the start of an argument. Only assertoric statements (ones that make a claim about the world) can be premises.

Conclusion: An argument's *conclusion* is the result of the logical argument

Valid: A logical argument is said to be *valid* if and only if it is impossible for the premises to be true and the conclusion false. Every argument where it is impossible for the premises to be true (i.e. if the premise must be false) is valid. Every argument where the conclusion must be true (it is impossible for it to be false) is valid. As we progress in this course, we will switch to a more sophisticated notion of validity, but this is fine for now.

Sound: A logical argument is said to be *sound* if and only if it is valid and the premises are true.

Exercises: For each of the following arguments, identify the premises and conclusion, and determine whether or not the argument is valid/sound.

1. John Boehner either is in the library or in the coffee shop. John Boehner isn't in the library, so he is in
the coffee shop.

2. Every state has two Senators. There are more members of the House of Representatives than Senators. Therefore, every state has more representatives in the House than the Senate.

3. All Toasters are made of wood. All objects made of wood are valuable. Therefore, all toasters are valuable.

4. We can be sure that humans are animals. After all, humans are mammals and all mammals are animals.

2 The Language of Propositional Logic:

Propositional logic has two kinds of symbols: letters for atomic sentences, and connectives that relate other sentences. Usually, sentence symbols begin with the letter p and progress alphabetically. If there are sufficiently many sentence symbols, you can distinguish them with subscripts. The goal in translating sentences to propositional logic is to capture as much detail as you can.

Negation: The symbol for negation in FOL is '$\neg$'. We standardly translate words like 'not' or 'un' to negations in FOL. So the sentence 'Megan is not tall' can be translated to: $\neg Tall(megan)$. Negation takes a single truth-value as input and has a single truth value as output. The truth table for negation is:

P	$\neg P$
TRUE	FALSE
FALSE	TRUE

Conjunction: The symbol for conjunction in FOL is '$\wedge$'. We standardly translate words like 'and' or 'but' to conjunction in FOL. So the sentence 'Grass is green and water is blue' can be translated to: $Green(grass) \wedge Blue(water)$. Conjunction takes two truth-values as input and has a single truth-value as output. The truth table for conjunction is:

P	Q	$P \wedge Q$
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	FALSE
FALSE	FALSE	FALSE

Disjunction: The symbol for disjunction in FOL is '$\vee$'. We standardly translate words like 'or' or 'either' to disjunction in FOL. So the sentence 'Either Mars is a planet or Earth is' can be translated to: $Planet(mars) \vee Planet(earth)$. Disjunction takes two truth-values as input and has a single truth-value as output. The truth table for disjunction is:

P	Q	$P \vee Q$
TRUE	TRUE	TRUE
TRUE	FALSE	TRUE
FALSE	TRUE	TRUE
FALSE	FALSE	FALSE

So a disjunction is only false when *both* disjuncts are false.

Exercises: For each of the following sentences in English construct compound sentences in propositional logic.

1. Either John is tall or short, but not both.

2. Martha is not to the right of Sarah.

3. Either Mike is both right-handed and a democrat, or Elizabeth likes cheese.

4. Although Rob is to the right of Max, he's to the left of Jane.

5. Margaret is both tall and a philosophy major.

Conditional: The symbol for the conditional in FOL is $\rightarrow$. We standardly translate words like 'if' and 'only if' to the conditional in FOL. Unlike conjunction and disjunction, the order of the sentences around the conditional matters: $A \rightarrow B$ is not equivalent to $B \rightarrow A$. 'If A then B' is translated to $A \rightarrow B$, 'A if B' is translated to $B \rightarrow A$ and 'A only if B' is translated to $A \rightarrow B$. The truth-table for the conditional is:

P	Q	$P \rightarrow Q$
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	TRUE
FALSE	FALSE	TRUE

Biconditional: The symbol for the biconditional in FOL is $\leftrightarrow$. We standardly translate phrases like 'if and only if' and 'just in case' to the biconditional in FOL. A biconditional is true just in case the sentences flanking it have the same truth-value; just in case they are either both true or both false.

P	Q	$P \leftrightarrow Q$
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	FALSE
FALSE	FALSE	TRUE

Exercises: For each of the following sentences in English construct compound sentences in propositional logic.

1. If John skips class, then both Sara and Anne will not be happy.

2. Max will major in math only if his mother tells him to.

3. If the stock market crashes, then either GM or Tesla will go out of business.

4. Jonah said that he will graduate only if he passes his logic exam.

5. Emily will have sushi for dinner if and only if Ralph does not.

6. Angela and Ahmad will both wear a coat only if it is either cold or raining.

7. If Cara either went to the supermarket or the library, then she will be late if and only if she did not check the time.

8. Rain causes the sidewalk to be wet.

9. Plato will not be a statesman only if Socrates drinks the hemlock.

10. It is not the case that if both Luke and Richard do not study, then the average score will be 90%.

11. If Ron and Harry both win 50 points, then if Hermione does not punch Malfoy, Gryffindor will either win first or second place in the house cup.

3 Truth Tables

Truth tables allow us to determine the situations in which complex sentences are true. We list all possible combinations of truth-values for sentence-symbols, and then use those to determine the truth-values of more complex sentences. This is going to resurface when we discuss model-theoretic conceptions of validity. A sentence is a *tautological truth* just in case it is true on every outcome of a truth-table. A sentence is a *tautological possibility* just in case there is at least one row on the truth-table where it is true. For example:

$$\neg P \vee (P \vee Q)$$

P	Q	$P \vee Q$	$\neg P$	$\neg P \vee (P \vee Q)$
TRUE	TRUE	TRUE	FALSE	TRUE
TRUE	FALSE	TRUE	FALSE	TRUE
FALSE	TRUE	TRUE	TRUE	TRUE
FALSE	FALSE	FALSE	TRUE	TRUE

Example: Fill in the following truth-tables. Which of the sentences are tautological truths, or tautological possibilities?

$$\neg P \vee \neg\neg Q$$

P	Q	$\neg P$	$\neg\neg Q$	$\neg P \vee \neg\neg Q$
TRUE	TRUE			
TRUE	FALSE			
FALSE	TRUE			
FALSE	FALSE			

$$(\neg(P \rightarrow Q)) \vee (\neg Q)$$

P	Q	$P \rightarrow Q$	$\neg(P \rightarrow Q)$	$\neg Q$	$(\neg(P \rightarrow Q)) \vee (\neg Q)$
TRUE	TRUE				
TRUE	FALSE				
FALSE	TRUE				
FALSE	FALSE				

$$(P \wedge Q) \vee (\neg P \wedge \neg Q)$$

P	Q	$P \wedge Q$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$
TRUE	TRUE					
TRUE	FALSE					
FALSE	TRUE					
FALSE	FALSE					

$$(P \vee Q) \rightarrow (P \wedge Q)$$

P	Q	$P \vee Q$	$P \wedge Q$	$(P \vee Q) \rightarrow (P \wedge Q)$
TRUE	TRUE			
TRUE	FALSE			
FALSE	TRUE			
FALSE	FALSE			

$$(((P \rightarrow Q) \rightarrow Q) \rightarrow Q)$$

P	Q	$P \rightarrow Q$	$(P \rightarrow Q) \rightarrow Q$	$((P \rightarrow Q) \rightarrow Q) \rightarrow Q$
TRUE	TRUE			
TRUE	FALSE			
FALSE	TRUE			
FALSE	FALSE			

$$(P \wedge Q) \vee ((\neg R) \leftrightarrow P)$$

P	Q	R	$P \wedge Q$	$\neg R$	$(\neg R) \leftrightarrow P$	$(P \wedge Q) \vee ((\neg R) \leftrightarrow P)$
TRUE	TRUE	TRUE				
TRUE	TRUE	FALSE				
TRUE	FALSE	TRUE				
TRUE	FALSE	FALSE				
FALSE	TRUE	TRUE				
FALSE	TRUE	FALSE				
FALSE	FALSE	TRUE				
FALSE	FALSE	FALSE				

We can also use truth-tables to determine when one sentence entails another. Sentence A *tautologically entails* sentence B just in case every line in a truth-table in which A is true is also a line in which B is true. We say that A and B are *tautologically equivalent* just in case A and B are true on precisely the same lines of a truth-table.

Exercises: For each of the following pairs of sentences, use truth-tables to determine whether the first tautologically entails the second, the second tautologically entails the first, the two are tautologically equivalent, or none of the above:

$$(P \wedge Q) \rightarrow \neg Q, P \wedge \neg Q$$

P	Q	
TRUE	TRUE	
TRUE	FALSE	
FALSE	TRUE	
FALSE	FALSE	

$$\neg P \rightarrow Q, \neg Q \rightarrow P$$

P	Q	
TRUE	TRUE	
TRUE	FALSE	
FALSE	TRUE	
FALSE	FALSE	

$$\neg(P \wedge Q), \neg P \vee \neg Q$$

P	Q	
TRUE	TRUE	
TRUE	FALSE	
FALSE	TRUE	
FALSE	FALSE	

Practice Problems

1. Identify the premises and conclusion in the following argument:

We can be sure that Saturn is the only planet with rings. After all, it is only possible for planets between 800-900 million miles from the sun to have rings, and Saturn is the only planet in that range.

2. Translate the following sentences into FOL:

a) Either Robert will marry Susan, or he will remain a bachelor as long as John does too.

b) If the Democrats don't nominate Warren, then either Sanders or Harris will win the nomination if Biden doesn't win.

3. Complete the following truth-tables and determine if the formulas are tuatologically possible or tautologically necessary.

$$\neg P \wedge \neg\neg P$$

P	$\neg P$	$\neg\neg P$	$\neg P \wedge \neg\neg P$
TRUE			
FALSE			

$$(P \wedge Q) \vee (\neg P \vee \neg Q)$$

P	Q	$P \wedge Q$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$(P \wedge Q) \vee (\neg P \vee \neg Q)$
TRUE	TRUE					
TRUE	FALSE					
FALSE	TRUE					
FALSE	FALSE					

4. For each of the following pairs of sentences, use truth-tables to determine whether the first sentence tautologically entails the second, the second tautologically entails the first, the two are tautologically equivalent, or none of the above.

$$(P \vee Q), (\neg Q \rightarrow \neg P)$$

P	Q	$P \wedge Q$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$(P \wedge Q) \vee (\neg P \vee \neg Q)$
TRUE	TRUE					
TRUE	FALSE					
FALSE	TRUE					
FALSE	FALSE					

$$\neg(P \rightarrow Q), \neg Q \wedge P$$

P	Q	$P \wedge Q$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$(P \wedge Q) \vee (\neg P \vee \neg Q)$
TRUE	TRUE					
TRUE	FALSE					
FALSE	TRUE					
FALSE	FALSE					